Administrative Council for Economic Defense
Department of Economic Studies of CADE

# Working Paper

Nº 006/2021

# New developments on market definition

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Brasília, December 2021

CADE
Conselho Administrativo de Defesa Econômica

# Ministry of Justice and Public Security
# Administrative Council for Economic Defense

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ISSN 2764-1031

This document was prepared by the Department of Economic Studies of CADE (DEE/CADE). It was translated from Portuguese into English by Arianne Mesquita Rodrigues, Ariel Daltrozo Munhoz Menezes, and Bruna Queiroz Assunção, in-house translators at CADE's International Unit.

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## EXECUTIVE SUMMARY

The Department of Economic Studies of the Administrative Council for Economic Defense (CADE) had the opportunity to write a working paper in 2010 summarising a few quantitative methods for market delimitation.

This document, in turn, aims to offer new perspectives on four methodologies:

(i) the critical loss analysis;

(ii) the aggregate diversion ratio methodology;

(iii) the use of simulations;

(iv) the assessment of counterfactual scenarios.

Briefly, this paper provides an updated list on quantitative methods for defining relevant markets.

It discusses topics such as simulations and counterfactual analyses, which were not included in the DEE's 2010 document. This paper also focuses on recent issues related to critical loss analysis, such as the debate about the interaction between intervals. In other words, the manner the antitrust authority should conduct reviews and determine whether it faces a set of standard deviations of price elasticity of demand—robustly assessed by econometric tests—which is supposed to interact with another set of values (interval of critical values). Additionally, Annex II presents the concept of "generalised critical loss", not addressed in 2010.

Similarly, we now introduce the concept of "diversion ratio" and "aggregate diversion ratio", showing in the Annex that the test may vary according to the circumstances. Thus, the aggregated diversion ratio approach varies depending on whether (i) we use the residual demand, (ii) the demand is sensitive to price rises, (iii) there is a spill-over to an adjacent market, (iv) there is an asymmetry between diversion ratios, (v) only one product suffers a price increase, (vi) amongst other issues that can influence the results. The aim is to indicate how the literature currently employs the cost pass-through rate and the aggregated diversion ratio to define relevant markets.

The previous working paper only mentioned one case in which the critical loss analysis was used—considering CADE had not fully explored the method yet. The present document, in turn, indicates many other cases in which the authority adopted this and other approaches, such as the aggregated diversion ratio and the counterfactual analysis. The efforts of the DEE have been instrumental in the adoption of these methodologies.

Therefore, this document presents a brief overview of this discussion within the Brazilian antitrust authority and of the need for updates.

**Keywords:** Relevant Market; Critical Loss; Aggregated Diversion Ratio; Simulation; Counterfactual Analysis; Cost Pass-Through Rate

## CONTENT

## 1. INTRODUCTION

The Department of Economic Studies (DEE) of the Administrative Council for Economic Defense (CADE) had already elaborated studies on methods of demand estimation[1], such as a working paper prepared in 2010 that summarised some quantitative means to delimit a relevant market.

Nonetheless, the present document aims to offer new perspectives on four types of methodologies:

(i) the critical loss analysis;

(ii) the aggregate diversion ratio methodology;

(iii) the use of simulations;

(iv) the assessment of counterfactual scenarios.

First, however, we should address the discussions on the validity of market delimitation methodologies.

In this regard, Kaplow (2010, p. 440) states:

> The central, conceptual argument is that there does not exist any coherent way to choose a relevant market without first formulating one's best assessment of market power, whereas the entire rationale for the market definition process is to enable an inference about market power. Why ever define markets when the only sensible way to do so presumes an answer to the very question that the method is designed to address? A market definition conclusion can never contain more or better information about market power than that used to define the market in the first place. Even worse, the inferences drawn from market shares in relevant markets generally contain less information and accordingly can generate erroneous legal conclusions — unless one adopts a purely results-oriented market definition stratagem under which one first determines the right legal answer and then announces a market definition that ratifies it.

According to this view, instead of defining a relevant market, an antitrust authority could, for instance, use simulations, counterfactual analysis, and other available means that do not use market share as an essential input for competition analysis. Other authors, such as Schmalensee (2009),

[1] Working Paper no. 001/2009, jointly prepared by the DEE and the Technical Group on Economic Methods (GTME), discussed demand analysis, including econometric methodologies to measure demand sensitivity. Moreover, Huse & Salvo (2006) made a didactic presentation about defining demand and supply parameters in a project developed by the Secretariat of Economic Law (SDE), the Institute of Applied Economic Research (IPEA), and the Brazilian Association of Graduate Programs in Economics (Anpec).

Shapiro & Farrell (2010) and Pakes (2010), also agree there are better alternatives to antitrust analysis that disregard market delimitation.

Conversely, Gregory Werden (2014) argues the relevant market still is a possible and necessary means for antitrust review.

Although we will not delve into this debate, we can say that the Brazilian antitrust authority still significantly employs the relevant market definition to adjudicate merger reviews and administrative proceedings through summary and ordinary procedures[2], as well as to analyse a transaction on its merits[3]. Moreover, non-compete clauses are limited to the scope of the relevant market in which a party operates (based on Precedent no. 4 published in the Brazilian Official Gazette on 9 December 2009).

Thus, although most modern authors are against the exclusive or primary use of said analytical means, the relevant market is still of the utmost importance to CADE.

According to Gama and Ruiz (2007), about 97% of CADE's decisions exclusively use *qualitative* information to define the relevant market. Besides, the case law that uses quantitative information does not necessarily have a clear or structured methodology for defining a relevant market.

From another point of view, the DEE has the technical expertise to carry out meticulous quantitative analyses—a duty performed since its establishment (DEE, 2010). This happens particularly when a case involves complex discussions.

The preference for defining markets only through qualitative means can hide a significant methodological problem since, from a qualitative perspective, there will always be differences amongst products; moreover, interpreting qualitative information in a generalised manner is not always possible.[4]

Hovenkamp (1999, p. 73) correctly mentioned that:

> . . . antitrust writers who are untrained in economics rely heavily on noneconomic values because this enables them to have an antitrust policy without undertaking the

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[2] According to Article 8, Item III of CADE Resolution no. 2/2012, drafted in Resolution no. 9/2014.

[3] See Article 8, Item V of CADE Resolution no. 2/2012.

[4] David Scheffman and Mary Coleman (2005, pp. 117-118) take the same view. As stressed by Neves, qualitative studies may contain numerous limitations, such as differences between the signifier-signified analysis: *"The gravest problem, however, seems to lie in the fact that the assessment methodology and conventions are not well established, unlike in quantitative research. Innovative, universal, and seemingly undeniable findings may in fact be wrong"* (Neves, 2006, p.4). This is true especially when solely based on qualitative methods, we may add. A qualitative-only market definition depends (more) on the analyst's own subjective perception of which market features are important to the concrete case in terms of substitutability.

(sometimes difficult) task of learning how the market system works. That approach may be easier in the short run, but it is calculated to have painful consequences in the long run.

The quantitative analysis alone can also result in problems, such as creating implausible or useless models. Thus, even if there is no precise method to delimit a market, it would be best to compare quantitative and qualitative means in a minimally structured way (or at least provide society with some consideration of this kind).

From a quantitative standpoint, this paper will not consider the price level, price correlation, and the geographic flow of customers and suppliers to define a market, as these have limitations the DEE (2010) already commented on and that are acknowledged by CADE.

For instance, in the Braskem/Solvay merger case, in 2014[5], former Commissioner Gilvandro V. Coelho de Araújo found issues in the tests of price correlation whereas the DEE, in Technical Note no. 7/2020 (doc. no. 0722205)[6], made strong criticism to a market definition based on the flow of products and customers.

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[5] Vol. 18 of the case records (doc. no. 0095138, p.295). For example, former Commissioner Gilvandro V. Coelho de Araujo (AC Braskem/Solvay, 2014) made the following remarks concerning price co-movement analyses: When carrying out a price co-movement analysis, the aim is to check price variance, i.e. whether price X varies in a "similar" manner to price Y. However, studies on the co-movement of prices are limited, as they do not control specific characteristics (as or even more relevant than price variance). Below there are some criticisms and necessary controls that should be considered in price co-movement analyses. **Firms' profitability** (an essential research tool that looks at competition level). This sort of assessment only happens in other types of tests (e.g. critical loss analyses). A test that measures price variation amongst products cannot ensure that, after a structural change in the market, this same variance level or price level will continue existing. A prognostic model capable of measuring that necessarily depends on measuring demand on both sides of players' profits. Only then does one know if increasing a price margin is profitable. This type of measure can be performed, for instance, through critical loss analysis but not via testing for the mere co-movement in prices. **Characteristics of the demand**. The analysis of the co-movement of prices does not examine the quantity variance (demand curve) as a reaction to price variance. Examining demand like this is relevant because it allows one to determine whether a possible correlation is due to price discipline. We stress that there may be false price correlations, influenced by the variance of inputs common to products X and Y (and not due to pressure from consumption). **Relevance of assessing price and margin levels**. If analysts do not assess price levels, they may wrongly gauge a market's rivalry level even with a similar variance, legitimate correlations, and control for inputs in common.

[6] Excerpt from the Technical Opinion no. 7/2020 in the scope of Merger 08700.002346/2019-85: Elzinga and Hogarty proposed consumer flow tests, which were named after them (ELZINGA & HOGARTY, 1973) (ELZINGA & HOGARTY, 1978). The authors assumed if there is trade between significant regions in terms of volume, the markets are probably integrated. A relevant market should have little imports from other regions (less than 10% for robust evidence or less than 25% for weaker evidence) and little exports (less than 10% or less than 25%, depending on the test). As widespread as the test is, there are several criticisms about its methodology. Firstly, researchers like KASERMAN & ZEISEL (1996) questioned the 10% and 25% values as they lack theoretical support and practical justification. STIGLER & SHERWIN (1985) also believe the presence or absence of product flow is not necessary nor sufficient to delimit a single geographic market. Indeed, the absence of product flow does not mean consumers will not resort to imports in the face of a substantial price increase. Geographically separated areas with high cross elasticity and no flow of goods may be deemed wrongly as distinct relevant markets through the Elzinga-Hogarty test. Elzinga himself acknowledged some limitations of the test (ELZINGA & SWISHER, 2011). EPSTEIN & RUBINFELD (2004) and MOTTA (2004) suggest the test tends to define excessively narrow markets as it does not examine

Even though techniques based on price comparisons and product and consumer flow can give insights on a market's competition level, they do not directly assess the interaction between prices and quantities nor do they enable a predictive analysis of players' profits. Hence, they are not enough to support the implementation of a hypothetical monopolist test.

Although less adopted, methodologies such as the critical loss analysis, aggregate diversion ratio, and counterfactual analysis and simulations seem more appropriate. Hence, to popularise these techniques, this paper intends to delve into them and show their relevance in antitrust debates by comparing them with other methods.

These methodologies (especially critical loss testing and ADR) are used in hypothetical monopoly tests (HMT), which examine whether a hypothetical monopolist can profitably impose a 5% price rise (as a rule),[7] considering a specific group of products/services within a geographically defined area. This is called a small but significant and non-transitory increase in price (SSNIP). According to this technique, the smallest market possible within which applicants' could impose a SSNIP is considered the ideal limit of a relevant market. [8] Goods (products or services) within a candidate market are sometimes called inside goods, whereas all other traded goods are named outside goods.

In case the candidate market is too narrow for a SSNIP, its scope should be increased (adding the closest substitutes in terms of product or geographic area) and tested once again if the SSNIP is possible. Hence, the relevant market gradually expands. The exercise only ends when one finds the smallest group of products and localities where the applicants' products exist in which the supposed monopolist can actually impose a SSNIP. This will define the relevant market in two dimensions: product and geographic. The HMT has limitations, especially when the expansion of the market does not add near substitutes.[9]

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what happens if there is a SSNIP and is merely founded on the current trade between regions (whose structural conditions change with the merger).

[7] In CADE's Guide for Horizontal Merger Review, the SSNIP is set at 5%. In the United States, the accepted SSNIP is also 5%, considering the competition authority reserves the right to set a higher or lower SSNIP according to the specificities of the analysed industry (see Section 4.1.3 of the US Horizontal Merger Guidelines). Werden (2014, p.6) claimed the US already took the position to adopt a 4% SSNIP. As stated by Ginevra Bruzzone (1995), the decision to impose a 5% SSNIP should not be made poorly. **US Horizontal Merger Guidelines, 2010** published by the FTC and the DOJ on 19 August 2010. Available at https://www.ftc.gov/sites/default/files/attachments/merger-review/100819hmg.pdf, accessed 14 March 2016. ***Guide for Horizontal Merger Review,*** originally published in the Official Gazette on 2 August 2016, Section 1, p.33), https://cdn.cade.gov.br/Portal/centrais-de-conteudo/publicacoes/guias-do-cade/Guide-for-Horizontal-Merger-Review.pdf, accessed 1 July 2019.

[8] We stress that the starting point for the HMT are the applicants' own products. This means mergers can have distinct starting points and, hence, distinct market definitions.

[9] That is, the rationale of an HMT is determining whether the starting point (i.e. where products A and B overlap) creates a SSNIP or not. Whenever the answer is no, the market can be further expanded. However, the HMT does not consider that a

Regarding the test, the US Horizontal Merger Guidelines (2010, p.9)[10] mentions: "The SSNIP is employed solely as a methodological tool for performing the hypothetical monopolist test; it is not a tolerance level for price increases resulting from a merger".[11]

The HMT has breakeven and profit-maximising versions (Baumann & Godek, 1995)[12].

- **Breakeven HMT:** it is sufficient that the profit increase offsets the demand lost with the SSNIP, which leaves the monopolist in the same profit situation before and after the SSNIP; or
- **Profit-maximising HMT**: in this case, in addition to offsetting the lost demand, the SSNIP is deemed possible if the profit after the price rise is greater than the profit before it.

The HMT can also use scenarios of kinked demand curve (concave or convex to the origin)[13] or even scenarios in which the hypothetical monopolisation creates efficiencies by reallocating

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starting point can be too broad, which can happen when there is no cross-price elasticity of demand between A and B (i.e. there is a diversion ratio of zero between the two products). Another problem exposed by Kate & Niels (2008) is that some products simply do not have near substitutes. Hence, if the hypothetical monopolist cannot impose a SSNIP, it is not enough to include substitutes in the market as it would not improve the merger review. The authors illustrate it with an example about cement. According to them: *"Cement has hardly any viable substitutes. When its price goes up, one may produce larger bricks to construct the same wall with less cement or one may start to build wooden houses; that is demand reduction, not substitution as such. There are no other materials available to stick sand together [in a similar manner]."* (Kate & Niels, 2008, p.318). Evidently, this observation depends on the technology available at the time; nonetheless, it can undoubtedly illustrate the problem antitrust authorities face. Furthermore, the HMT (at least in its traditional form) makes some assumptions, such as that the monopolist does not own companies in adjacent markets (i.e. markets where there are close substitute products that consumers would start to take into account or purchase in case of a SSNIP). To be adjacent to the analysed market, it is enough that the cross elasticity between the two markets is different from zero. If the monopolist operates in a market that is adjacent to the one it already monopolises, the HMT will be flawed since the monopolist can recover part of the demand lost with the SSNIP. Moreover, it is important that CADE knows whether the merger applicants are in adjacent markets because this may influence the market definition and the potential effects of the merger. (Ivaldi & Lörincz, 2005).

[10] US Horizontal Merger Guidelines, prepared by the FTC and the DOJ on 19 August 2010. Available at https://www.ftc.gov/sites/default/files/attachments/merger-review/100819hmg.pdf, accessed 14 March 2016.

[11] The guides for horizontal merger review published in 2001 by the Secretariat of Competition Advocacy and Competitiveness (SEAE) and in 2016 by CADE found inspiration in the US position on the HMT, adopting the SSNIP to delimit a relevant market. *Guia de Análise de ACs horizontal*, published in the Official Gazette no. 158-E on 17 August 2001, Section 1, pp. 12–15, according to Joint Ordinance SEAE/SDE no. 50. *Guide for Horizontal Merger Review*, published in the Official Gazette on August 2 2016, Section 1, p.33), https://cdn.cade.gov.br/Portal/centrais-de-conteudo/publicacoes/guias-do-cade/Guide-for-Horizontal-Merger-Review.pdf, accessed 1 July 2019.

[12] According to D'Amore & Mercuri (2003), as per Resolution 164/2001 of the former Secretaria de la Competencia, la Desregulación y la Defensa del Consumidor, Argentina adopts the breakeven criteria to define a market. The US Horizontal Merger Guidelines (Section 4.1.1), in turn, prefers the profit-maximising HMT via a single-product SSNIP but recognises that sometimes the applicants present breakeven measures that can be used (see Section 4.1.3 of the guidelines).

[13] In this respect, we stress there are cases in which a 5% price rise is not profitable but maybe 10% or 15% are. According to Gregory Werden (2002), it is quite usual that for consumer groups of certain products, demand elasticity varies greatly in relation to price along the demand curve. As the author puts it, some reasons for this are that there is more than one manner and more than one place to use a product. Hence, it is possible to have a kink in the demand curve. We stress, however, that this depends on whether the curve is concave or convex to the origin. The curve Werden cites, which could justify an attempt

production amongst the monopolist's plants with distinct idleness levels (as seen in Gregory Werden, 2002).

With this brief introduction, we will next describe the structure of this paper. Firstly, we present quantitative methodologies (critical loss, aggregate diversion ratio, and counterfactual simulations and analysis); next, we look at how CADE has addressed these topics in its case law, if at all. Finally, we present challenges intrinsic to adopting the concept of relevant market, e.g. its use for multi-sided markets.

## 2. QUANTITATIVE METHODOLOGIES

On several occasions, the relevant market is defined by assessing demand-side substitution. In this regard, we could mention *Columbia Metal Culvert Co. v. Kaiser Aluminum & Chem. Corp.*, in which the court understood it is the perspective of consumers—and not producers—that should be deemed when delimiting a relevant market. Arguing the opposite, many precedents consider that a market delimitation only from the demand side is insufficient and should include the concept of price elasticity of supply (such as *Rebel Oil Co. v. Atlantic Richfiel Co.,* adjudicated by the 9th Circuit).

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towards a SSNIP over 5%, would be a kinked demand curve that is convex to the origin. Werden mentions *United States v. Archer-Daniels-Midland Co., 1988*, in which a small price rise would lead part of the consumers that replaced high-fructose syrup for sugar to turn back to sugar. However, greater price increases would target fewer elastic consumers that could not make this substitution as spontaneously and automatically. CADE has already faced debates over claims of kinked demand curves concave to the origin, analysing how it could impact the HMT. The discussion happened in the context of the Petrobrás/Ipiranga/Braskem/Quattor merger, 2008. In this case, the authority tried to assess whether the geographic market of some thermoplastic resins had a national or international scope. The applicants argued the demand curve kink happened exactly where the national price was equal to or greater than the international one. According to the allegations in the case record: "*a kinked demand curve is expected in a market that is integrated into the international market. In a small country, the residual demand curve faced by the country's producers should be very elastic in the branch in which the national price is equal to or greater than the international price. Even if integrated into the international market, there may be periods in which the national price drops below the international price. In these events, the residual demand becomes less elastic, as domestic consumers no longer have the option to substitute national products at the same price level. In this interval, the price elasticity comes to represent only the substitution of the product by others".* However, this type of claim may indicate the opposite: that the domestic competition is so relevant that when national prices are slightly lower than international prices, consumers modify their consumption behaviour (to the extent of changing the price elasticity of demand). That is, when faced with the slightest price difference between the national and international version of a product, consumers end up preferring the national one—due to inherent characteristics such as quick delivery, post-sale service, amongst others. Furthermore, when a merger potentially creates concentration in the domestic market (which has the characteristics above), a 5% SSNIP may be exceedingly high, especially when the concentration can create a non-hypothetical monopoly or duopoly in the domestic market. In such cases, the SSNIP may function almost like a simulation of the effects of a merger (as national consumers prefer the domestic product). In any event, if there is a kinked demand curve, it should be computed in measuring the actual loss the monopolist would face if they attempted a SSNIP. However, one should not use solely the greatest or smallest elasticity to calculate the actual loss but rather heed the price level and its distance from the kink to estimate the hypothetical monopolist's actual loss.

According to Atilano Jorge Padilla, PhD,[14] the Court of Justice of the European Union established the importance of supply-side substitution in the *Continental Can* [15]and *Michelin*[16] cases. In Brazil and in the US, researchers have reservations about examining supply-side substitution to define markets.[17] Evidently, it would be necessary to formulate a test that determines how large supply substitutability should be to constrain a hypothetical monopolist. This does not seem to be well established in the academic debate, although, as we later discuss, delimiting a market via FERM testing may address some of these issues.

Nonetheless, we do not intend to address the merits of this discussion but only mention that the methodologies presented below focus on demand-side substitution.

### a. Critical loss analysis

Harris & Simons (1989) wrote a paper on how to implement the hypothetical monopolist test through a method called critical loss. This technique compares a monopolist's actual demand loss after a SSNIP against the critical (or maximum) loss the monopolist could incur to make a profit or at least break even. If the actual loss is greater than the critical loss, the relevant market should be expanded. Conversely, if the actual loss is lower than the critical loss, the relevant market is defined properly. Below are the critical values for the demand lost after a SSNIP:

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[14] According to https://pdfs.semanticscholar.org/57f5/5e6f2405a22b7ce8469a0e6efb0bc9b6ae4f.pdf?_ga=2.200945446.902426088.1587120098-343898306.1587120098, accessed 17 April 2020, *"The European Court of Justice clearly established the importance of incorporating supply-side substitutability considerations at the market definition stage in Continental Can. The Court rejected the definition held by the Commission in this case on the grounds that it failed to consider substitutes on the supply-side. The Court reaffirmed its position on this issue in later cases (such as, for example, in Michelin). The Commission's Notice took account of the Court's opinion on this matter and explicitly introduced consideration of supply-side substitutability as part of its approach to market definition."*

[15]Continental Can, Case 85/76 [1972] ECR 215; [1973] CMLR 1999; [1972] OJ L 7/25.

[16] Michelin, Case 322/81 [1983] ECR 3461.

[17] The definition of the relevant market considering supply-side substitutability is related to the assessment of the capacity and availability of other firms to start producing and supplying the product at issue in a said area, after a small but significant and non-transitory increase in price (SSNIP) and in a short length of time. In other words, it observes whether, given an increase in the sales price, other firms can offer the good in the same geographic market applicants operate. The assessment of supply-side substitutability is the same one involved in the barriers-to-entry analysis (see item 2.5.1), with some specifications: (i) products must be supplied in less than a year, (ii) without it being necessary to incur in sunk costs. Following international practices, CADE favours defining the relevant market considering the demand and understands considerations involving the supply-side to be an additional factor in assessing the effects of the transaction. *Guide for Horizontal Merger Review*, published in the Official Gazette on August 2 2016, Section 1, p.33,https://cdn.cade.gov.br/Portal/centrais-de-conteudo/publicacoes/guias-do-cade/Guide-for-Horizontal-Merger-Review.pdf, accessed 1 July 2019.

**Table 1 - Critical loss**

| Demand curve | Profit maximisation | Breakeven |
|---|---|---|
| Linear | $\frac{s}{(m + 2s)}$ | $\frac{s}{(m + s)}$ |
| Isoelastic | $1 - (1 + x)^{\frac{-1-s}{m+s}}$ | $\frac{s}{(m + s)}$ |

**Source:** Werden (2002) (1998) Baumann & Godek (1995)
s = SSNIP
m = price-cost margin (pre-SSNIP)

The test may also be implemented by comparing the actual elasticities of demand with the critical elasticities. In case the actual elasticity of demand (in absolute value) is greater than the critical elasticity (in absolute value), the relevant market should be expanded as the SSNIP is unprofitable.. If that is not the case, the market is properly delimited.

**Table 2 - Critical elasticities**

| Demand curve | Profit maximisation | Breakeven |
|---|---|---|
| Linear | $\frac{1}{(m + 2s)}$ | $\frac{1}{(m + s)}$ |
| Isoelastic | $\frac{(1 + s)}{(m + s)}$ | $\frac{\log(m + s) - \log(m)}{\log(1 + s)}$ |

**Source:** Werden (2002) (1998) Baumann & Godek (1995)
s = SSNIP
m = price-cost margin (pre-SSNIP)

The table below exemplifies how critical loss can be applied in a specific case:

- Suppose a hypothetic monopolist in a market sells 1,000 units of a certain product at $1,000, with a profit margin of 10% (that is, a profit per unit of $100). In this case, the total profit of said firm will be $100,000.

- Should this firm increase the price by 5% (the percentage authorities usually regard as a SSNIP), and consumers do not react to the increase but continue to buy the product, the player's profit will go from $100,000 to $150,000.

- In the real world, however[18], what usually happens is that as the price goes up, consumers buy less. Therefore, to determine whether a SSNIP is profitable, it is essential to know how many consumers stop buying a product after the SSNIP. In this

[18] Except for specific hypothesis of Giffen goods.

specific example, should the monopolist wish to maintain that profit, they can lose no more than 33.33% of its initial demand, considering a linear demand. If the lost demand is greater than this, the SSNIP will not be profitable—which, hence, requires expanding the relevant market.

**Table 3 - Example: HMT via critical loss**

| *Formula* | *Variable* | *Value* | *SSNIP without demand loss* | *SSNIP with demand loss up to the critical value of the 33.3% linear break-even** |
|---|---|---|---|---|
| Q | Quantity sold | 1,000 | 1000 | 666.66...* |
| P | Price per unit | $ 1,000 | $ 1,050 | $ 1,050 |
| L | Profit per unit | 100 | 150 | 150 |
| T=L*Q | Total profit | $100,000 | $ 150,000 | $ 100,000 |
| M=L/P | Percentage profit =0.1 | 10% | | |
| S | SSNIP = 0.05 | 5% | | |
| CL=S/(S+M) | Critical loss=0.05/(0.05+0.1)=0.333... * | 33.3...%* | | |
| CE=1/(S+M) | Critical elasticity=1/(0.05+0.1)=6.66...* | 6.66... * | | |

***Recurring decimal**

Thus, it is important to estimate players' elasticity and profit to define a market.

In the example above, if after the SSNIP the company can maintain the sale of only 666 products, it will make a loss (as it will not sell 334 products or 33.4%, which is greater than the critical value of 33.33...%):

- 666 * 150 = $ 99,900 = lower than the $100.000 pre-SSNIP profit.

Nonetheless, if the company sells only 668 products after the SSNIP (losing 332 products), it will **make a profit**, as the loss will be 33.2% (i.e. lower than the critical value of 33.33...%):

- 668 * 150 = $ 100,200 = greater than the $100,000 pre-SSNIP profit.

In this last hypothesis, in which the hypothetical monopolist profits after the SSNIP, the market is well delimited since, theoretically, the monopolisation allows for a price rise and the exercise of market power, other things being equal. This conclusion (that 33.3% is the maximum loss for a hypothetical monopolist to profit in the example above) depends on adopting the above-mentioned formulas:

**Equation 1 - Linear break-even critical loss with a 5% SSNIP and a 10% margin**

$$Critical\ loss = \frac{s}{s+m} = \frac{0.05}{0.05+0.10} = \frac{0.05}{0.15} = 0.33\ldots * = 33.3\ldots\% *$$

***Recurring decimal**

s = SSNIP
m = price-cost margin (pre-SSNIP)

Nevertheless, with this methodology, the applicants have incentives to claim high profit margins because as "m" increases in the formula above (and, hence, profit margins), the critical loss decreases. This encourages allegations that the relevant market is broader than the candidate market and that applicants do not exercise market power as their market share is diluted in an overall scenario. It furthermore implies a contradiction as higher profit margins translate into market power.

Therefore, several authors are sceptical about critical loss testing, arguing the competition authority should be extra careful in conducting the test, especially when informed about high profit margins in the analysed sector (Danger & Frech III, 2001), (Katz & Shapiro, 2003), (O'Brien & Wickelgren, 2003). This kind of criticism gives rise to a debate on whether an authority should use aggregate diversion ratio in its analysis, which we further explore later in this paper.

Next, we comment on how to calculate critical loss.

### i. Cost variable applied to calculate the profit margin of critical loss

To Michael G. Baumann and Paul E. Godek, the formulas to calculate critical values only make sense if they are based on the actual marginal cost. However, unlike Fisher (1987) and Baker & Bresnahan (2008), some researchers advocate for using the average variable cost (AVC) in formulas for critical values.

- **Using the AVC as a proxy.** Barry Harris (2015) theorises the possibility of adopting the average variable cost as a proxy for actual costs saved with the **decrease in sales.**[19]
- **Criticism against the use of AVC as a proxy.** Conversely, Carlton & Perloff (1999) contend that using the average variable cost may hugely and severely bias the analysis. Baker & Bresnahan (2008, p.19) argue that accounting data creates problems for several reasons:

[19] *"Marginal cost normally cannot be measured at all, but rather only proxied for by average variable cost. This typically is a legitimate practice if marginal cost is roughly constant, which commonly is the case. Nevertheless, a measure of actual production costs, even the incremental cost of producing the last unit, may not be a valid indication of economic marginal cost. The relevant cost concept is opportunity cost. The opportunity cost of a variable input may exceed its accounting cost if, due to scarcity, it is valued in alternative uses at more than its accounting cost. A capacity constraint creates the same phenomenon for fixed inputs, unless the constraint ceases to bind when output is restricted through the exercise of market power."* (Werden, 1998, p. 394).

difficulties in addressing depreciation and opportunity costs, the fact that marginal costs sometimes have a horizontal curve, etc.

Authors such as Coate & Williams (2005) agree that using AVC as a proxy for the marginal cost poses some problems. Due to this, both authors developed a concept known as generalised critical loss analysis. The technique is described in Annex II of this paper.

### ii. Interaction between intervals

Different cost measures can give a different profit margin and, consequently, a different critical elasticity. Thus, it is difficult to know, in theory, which critical elasticity formula fits best.

Additionally, variables are not self-evident.

Depending on how one calculates them, the "m" of the critical loss formula can be the Lerner index. Indeed, inspired by the work of Chamberlin (1933) and Robinson (1933), Abba Lerner (1934) developed a largely-adopted concept in measuring market power:

**Equation 2 - Average Lerner index for a given period in a given market**

$$Average\ Lerner\ index = \frac{Average\ Price - Average\ Marginal\ Cost}{Average\ Price}$$

On the other hand, although seemingly simple, the average Lerner index contains much information:

- The price may vary with the customer, the category and qualities of a product, the timeframe, etc. Hence, the average Lerner index is a type of profit margin calculated according to an average price, with a specific distribution and variation. With a striking price variation, the average Lerner index may not accurately represent the pricing decisions of a monopolist (however hypothetical). This can interfere in the concept of "critical elasticity". For instance, from a geographic perspective, prices that vary too much amongst places, or products with markedly different prices, indicate niche markets. The latter can only be identified after exploratory research, plotting prices on maps or graphs. In any case, this influences the assessment of a market's profit distribution, used to measure the "average price".

- As to marginal costs, they can be estimated through econometric analyses, in which one must decide what algorithm and oligopoly model to adopt, how to approach endogeneity (if at all), etc. In addition, there is the problem of the quality and quantity of information available and the limited time to review the merger and adjudicate the case. Moreover, (i) some companies may not know how to properly measure their own costs or not keep track of this information, (ii) they may have distinct production costs for different products and plants, which make for distinct Lerner indexes, and (iii) their production costs may vary over time, which can create a distribution of Lerner indexes over the analysed period. There may be some disagreements on fixed, variable, and marginal costs.

- Fisher (1987) discusses the complexity of estimating the Lerner index based on accounting costs. Conversely, if in addition to accounting costs we include economic costs, the marginal costs will become more complex as they allow for numerous opportunity costs.

- Furthermore, an econometric estimate of these costs leads to a distribution of the possible values (within a Bayesian credible interval or a confidence interval inference).

- If there is a distribution of price values and of marginal cost values, it also means there is a distribution of valid Lerner index values. Hence, the Lerner index would not be a point, but a distribution. Consequently, there is a distribution of critical elasticities.
- Thus, we should rather talk about a "Lerner interval". The Lerner interval denotes the distribution of combinations of the distinct prices and products directed at distinct customers in distinct timeframes, with different production costs (which may also vary depending on customers, timeframes, and products). This distribution of values also creates an interval of different critical values (critical elasticities).

If there is an interval of critical values, consequently there is a measurable interval (or distribution) of elasticities. In regression analysis, the result is an interval of results: a Bayesian credible interval or a confidence interval inference. This interval is given by a single regression. However, for the purpose of robustness testing, it is possible and advisable to perform several (sometimes thousands) regressions to assess a single price elasticity of demand (or distribution of elasticity).

These thousand regressions are required because, as seen above, there may be numerous combinations to address endogeneity concerns and different explanatory oligopoly models for a single market, with distinct functional forms to understand consumer behaviour towards price rises.

Hence, a simple HM test includes the fact that we cannot compare a critical value with an actual value. In reality, there is a comparison and an interaction between a "distribution of critical values of price elasticity of demand" and a "distribution of actual values of price elasticity of demand". It is the interaction of these distributions of intervals that the competition authority uses to delimit a relevant market.

### iii. Comparing critical and actual values

After measuring the critical and actual values, the competition authority uses them to assess the hypotheses of whether the market has been properly determined or not.

In some (rare) occasions, one of the hypotheses is clearly more probable.

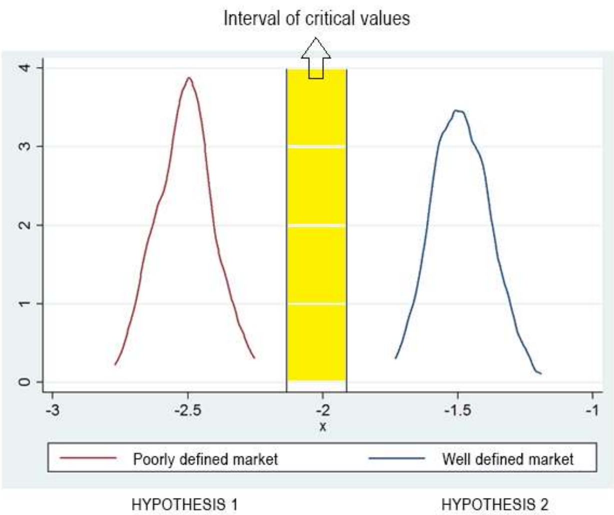
Consider the following:

- there is an "interval of critical elasticities" between -1.9 and -2.1 (yellow).

- this interval will be compared to two intervals of the measured elasticities:

- HYPOTHESIS 1 (the interval of actual elasticities is from -2.7 to -2.3, red curve). If the interval of all estimated models is greater (in absolute value) than the confidence interval/credible interval of critical values (in absolute value), the **relevant market is not properly defined** as the hypothetical monopolist would lose much of its demand by not profiting with the SSNIP, in all analysed scenarios.
- HYPOTHESIS 2 (the interval of actual elasticities is from -1.7 to -1.3, blue curve). If the interval of all estimated models is less (in absolute value) than the confidence interval/interval of critical values (in absolute value), the **relevant market is well defined**, since the hypothetical monopolist will lose little demand by profiting with the SSNIP, in all analysed scenarios.

**Figure 1 - One hypothesis clearly more probable than the other**

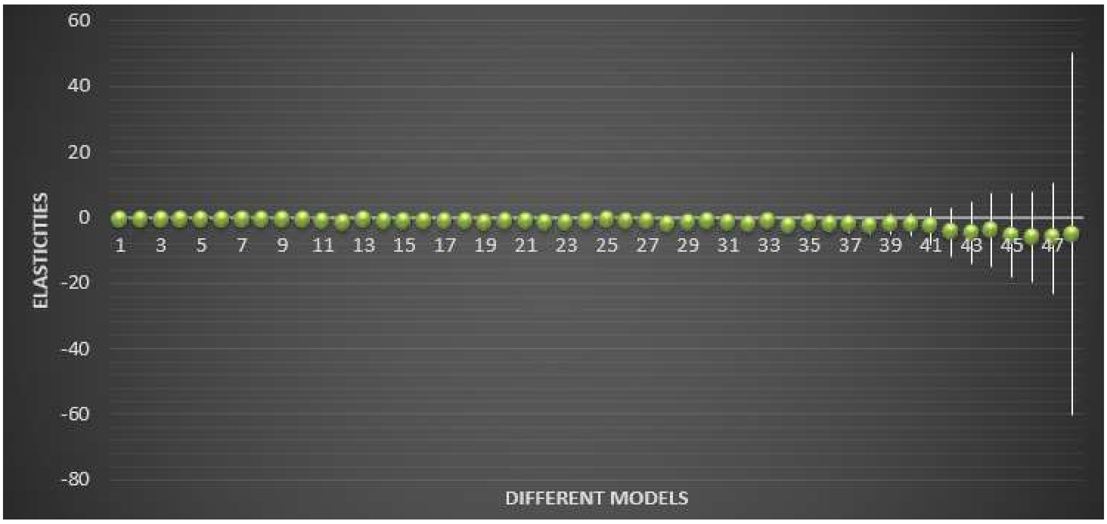



**Source:** CADE

However, it is not always that a hypothesis is explicitly more probable. Sometimes the elasticity values are too wide (and the interval cuts a critical value). One extreme instance of that happened in the Braskem/Solvay merger (2014). In this merger case, the plaintiffs maintained that the price elasticities of demand ranged from +50 to -60 for some of the used regressions. That is, they argued the elasticity of demand could go from the extreme positive elasticity to the extreme negative elasticity. This is to say that—according to the econometric model—the ambient temperature is between +1000 °C and -1000 °C, which does not mean much (and should not be an argument in favour or against a given market definition). However, the plaintiffs alleged that, as the actual elasticity was not known, the merger should be cleared. Former Commissioner Gilvandro V. Coelho de Araújo, rapporteur of the case, understood the applicants should not benefit from "the creation of models that, due to plaintiffs' inclusion of undue variables, jeopardise the predictive quality of elasticity, especially by means of multicollinearity".

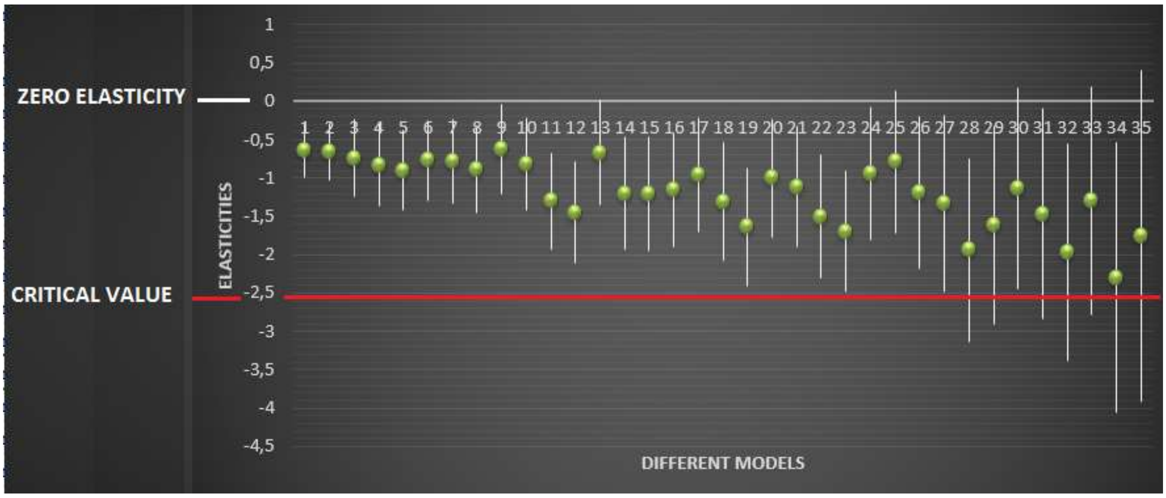
The argument used in the Braskem/Solvay merger (2014) is illustrative. More than 20 models had significant values (i.e. above zero) with the hypothesis of a narrow market being much more probable (as the entire measured interval was smaller than the critical value in absolute value):

**Figure 2 - All models presented by the plaintiffs**




**Green dots represent the average elasticity ** White lines represent the confidence interval***The elasticities are not in absolute value***Source:** CADE

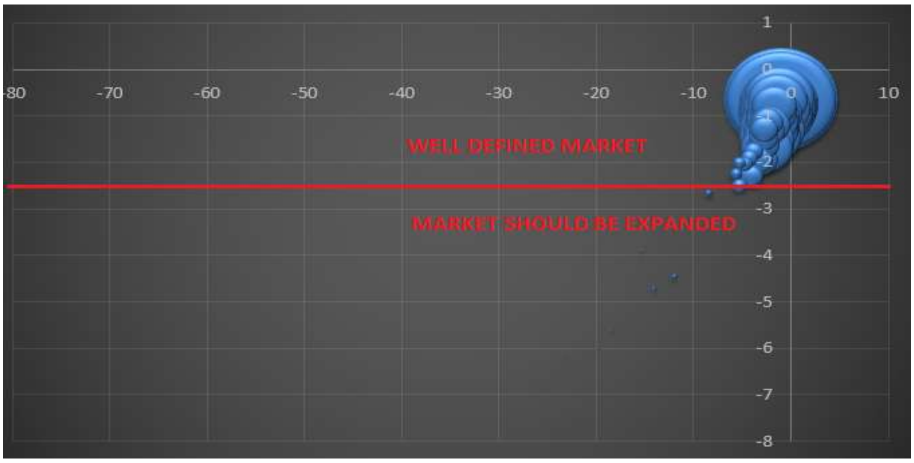
**Figure 3 - Thirty-five models with the measured intervals of price elasticities of demand lower than the critical elasticity (in absolute value)**




** Green dots represent the average elasticity*
*** White lines represent the confidence interval*
**** The red line represents the critical elasticity*
***** The elasticities are not in absolute value*
**Source:** CADE

Hence, in the Braskem/Solvay merger (2014), it was evident that the standard deviation diminished as the models reached a certain number, creating a convergence.

**Figure 4 - Convergence process (robustness analysis)**

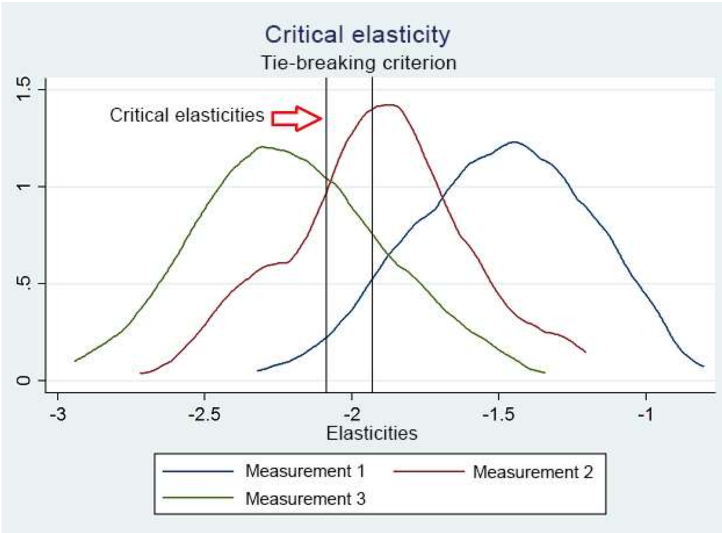



**Horizontal axis:** lower limit of the elasticity confidence interval.
**Vertical axis:** average elasticity
Radius of the circle = (standard deviation)$^2$. Hence, good models will have a greater area and, consequently, weight heavier in the chart.
**Source:** CADE

Therefore, the Department of Economic Studies presented its own elasticity models, with a lower standard deviation. Nonetheless, the exercise shows how in some cases one can pick a model or value over the other. Evidently, we should consider not only the standard deviation but several other factors and indexes such as the Sargan-Hansen and Stock-Yogo tests, all the assumptions for the model according to the adopted stochastic process, amongst others.

However, the measured elasticities may cut the mean of the critical elasticity intervals as in the graph below (with no hypothesis being clearly more probable than the other). If this is the case, we apply a tie-breaking criterion:

**Figure 5 - Estimated elasticities cut the critical elasticity or elasticities**




The tie-breaking criterion, in econometric terms, is the following. The one with the null hypothesis "wins" in case of a "tie", that is, the hypothesis is not disregarded. It can be rejected but does not need to be proved. In legal terms, whoever has the alternative hypothesis has the burden of proof. If one has the null hypothesis, nothing needs proving since, when in doubt, the hypothesis will not be rejected. Hence, we should ask: how to attribute the null hypothesis? There are two options, at least:

**Equation 3 - Precautionary principle: the narrower market has the null hypothesis**

$$H_o = |E| \leq |E_{CR}|; H_1 = |E| > |E_{CR}|$$

(if in doubt, the market is narrow)

**Equation 4 - Self-restraining principle: the broader market has the null hypothesis**

$$H_o = |E| \geq |E_{CR}|; H_1 = |E| < |E_{CR}|$$

(if in doubt, the market is broad)

$* \, |E| = confidence\ interval\ of\ the\ absolute\ value\ of\ the\ own - price\ elasticity\ of\ demand$
$** \, |E_{CR}| = absolute\ value(s) of\ critical\ elasticity(ies)$

In the analysis of the Braskem/Solvay merger (2014), CADE opted for using the precautionary principle.[20] This was much due to the fact that the applicants could present non-informative models

[20] Vol. 18 of the case records (doc. no. 0095138), p. 323, Paragraph 193 of the vote of former Commissioner Gilvandro. See also Paragraph 4 of the vote of former Commissioner Ana Frazão. Vol. 18 of the case records (doc. no. 0095138), p. 382.

for their own benefit[21], increasing the scope of the relevant market and allowing for an undue merger clearance.

### b. Methodology of the aggregate diversion ratio

Katz & Shapiro (2003) showed us that the critical loss analysis may have some flaws. They noted that, by using the critical loss test, merger applicants intend to claim high profit margins ("m") since with critical loss they can probably extend the relevant market, which makes their market power seem smaller and increases their chances of merger clearance.

In this context, the authors stated that "when gross [profit] margins are large, defence claims that the elasticity of demand is high should be treated with a healthy dose of scepticism. *More specifically, we advocate an approach under which there is a presumption that high gross margins go along with a low elasticity of demand faced by the hypothetical monopolist. . . .* In particular, if (before the merger) a firm chooses a high margin on its product, the firm evidently thinks that demand for its product is not very sensitive to price." (KATZ & SHAPIRO, 2003, P.52). O'Brien and Wickelgren (2003) also mention problems in the conception of the critical elasticity test. They consider critical loss analysis has two main flaws: (i) it does not recognise that firms' profit margins inform about elasticity of demand and (ii) it ignores the substitutability, i.e. the cross-elasticity of products subject to the SSNIP. [22]

The authors explain that the reason for exceedingly high pre-merger margins is that consumers are not very sensitive to price changes.

[21] With excessive multicollinearity, for instance.

[22] According to O'Brien and Wickelgren (2003)*: "Unfortunately, the standard way that critical loss analysis is applied (henceforth, 'standard critical loss analysis') ignores two key points, often leading to inconsistent logic and erroneous conclusions. First, standard critical loss analysis fails to recognize that a firm's margin provides information about the magnitude of the sales it is likely to lose from a price increase. If pre-merger prices are chosen to maximize profits, higher margins typically imply that customers are not very price sensitive (otherwise, a firm could substantially increase its sales by making a small price cut, which would imply that the current price is not profit-maximizing). This means that when margins are larger, a price increase will usually result in fewer lost sales than when margins are smaller. In short, the argument that a large percentage of sales would be lost in the event of a price increase is typically inconsistent with the existence of large pre-merger margins for profit maximizing firms. A second fundamental error of the standard critical loss analysis is that it ignores the importance of the degree of substitutability (e.g., the cross elasticities of demand or diversion ratios) among the products of the firm considering the price increase. The greater the cross elasticities, the more the firm will profit from increasing the price of one product because it will capture a larger percentage of the lost sales through increases in the sales of its other products. As an extreme example, consider a merger that would combine two products that have zero or very low cross elasticities between each other. A post-merger price increase on one of the products would not significantly raise the sales of the other, so the merger provides little or no incentive to raise price. On the other hand, suppose the merging firms have very high cross elasticities between each other. In this case, a price increase for one of the merging products results in substantial sales diverted to the other product, increasing the merged firm's profits. If margins are high, so that the diverted sales are highly profitable, the merged firm will have a relatively higher incentive to raise price absent offsetting entry, product repositioning, or efficiency gains."*

Scheffman and Simons (2003) disagree. They believe that not incorporating in any way the Lerner index nor the models of Bertrand or Cournot is an asset for the critical loss methodology and that we should not have preconceptions on how players price or profit:

> The significance and generality of CLA [i.e. critical loss analysis] lies in its ease of practical application and from the fact that it is merely "arithmetic" based on the assumption that a rational firm will not change price unless it expects, as a result, that its profits will not fall. CLA is independent of any particular theoretical model of pricing. . . . One of the greatest strengths of CLA is that it is "just arithmetic," and completely neutral as to the appropriate theoretic model that best explains any real-life market. In our experience, perhaps because of equations and diagrams in various papers discussing CLA, some merger analysts mistakenly believe that CLA is based on economic theory, beyond the simple assumption of profit-maximization. . . . We provide this example to make clear that CL Analysis does not involve economics equations or diagrams—it is just arithmetic. Suppose that you are a middleman who resells widgets. You purchase widgets for $5 a piece and you have no other costs of reselling widgets. Suppose you are able to resell widgets at retail at $10, i.e., you make a margin of $5 per widget, and you are currently selling 100 units. Suppose, finally, that you have not changed the retail price for quite awhile, and you are thinking about raising the retail price by $1 to $11. Would that raise your profits? Obviously, if your sales would remain at 100 units, the $1 price increase would increase your profits by $100. However, it is likely that at least some retail widget customers will reduce their purchases in the short and/or long run as a result of the 10 per-cent price increase. Given your prices and margins, the extent of sales lost due to the $1 price increase determines the impact of the price increase on profits. Suppose, for example, you lose sales of 10 widgets when you increase the price by $1. Profits before the price increase were $500 ($5 per unit for 100 units). After the price increase, profits will be $540 ($6 per unit for 90 units). In this case the price increase of $1 would increase profits by $40. However, notice that if sales fell to 83 units or lower, the price increase would lower profits to $498 or less. Note that we need no equations or graphs, and there are no elements of economic theory in the calculation. It is simply arithmetic. (Sheffman & Simons, 2003)

As to the perspective of Scheffman & Simons (2003), Farrell & Shapiro (2008) say one can have two reactions to this phenomenon:

- Stress the pre-merger evidence, assuming the firms maximised their prices and priced according to the price elasticity of demand.
- Ignore the pre-merger evidence and measure demand sensitivity instead of inferring it. However, if the demand sensitivity does not seem to match firms' pre-merger prices and margins, one should not worry.

The authors advocate for the use of pre-merger evidence, based on their scepticism towards critical loss analyses. They suggest deeming the ADR (aggregate diversion ratio) as some kind of test or pre-merger evidence adopted to define the relevant market[23], complementing the critical loss test.

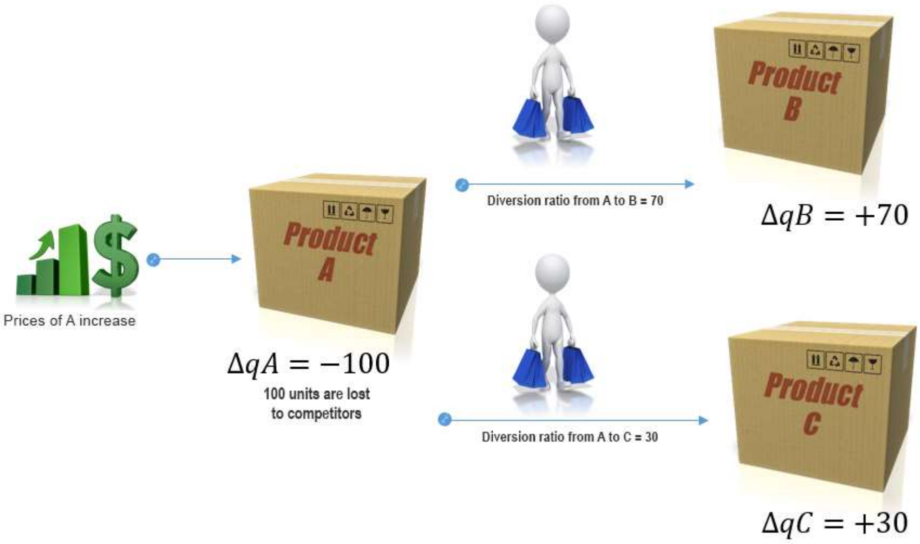
### i. Notions of Diversion Ratio

Before explaining aggregate diversion ratio (ADR), we should first describe the simple diversion ratio (DR). The term diversion ratio measures the substitution or competition between two or more products, which may translate into an important piece of evidence in merger reviews.

The diversion ratio measures the proportion of sales of different substitute products, in terms of quantities or revenue, when the price of another product increases. For instance, consider the products A, B, and C and suppose that, after a small increase in the price of A, its sales decrease in 100 units. As a consequence, products B and C capture these sales, with 70 of units diverting to B and 30 units to C. Therefore, the diversion ratio from A to B is 70 units (or 70%) and from A to C is 30 units (or 30%).

**Figure 6 - Diversion ratio between products**

[23] Some papers written before the development of Shapiro's technique did not believe in using cross elasticities to determine a market. Against this background, Werden (1998, p.380) considers that, to prove market power, many economists (and legal experts) resort to own-price elasticity of demand (as seen in Eastman Kodak C. v. Image Technical Servs. Inc 540 US (1992); Allen-Myland Inc. v. IBM Corp. (3rd. Circuit, 1994); United States v. Baker Hughers Inc (DC Circ., 1990); Ball Mem´l Hosp. Inc. v. Mutual Hosp. Ins. (7o. Circ. 1986). However, with regard to cross elasticity, Werden stated the first substantial initiative to determine a market based on these variable dates from 1952 (citing Joe Bain and Fritz Machlup). The author clarified there were cases that used cross elasticity in market definition, such as in United States v. E.I du Pont de Nemours & Co. (1957); Brown Shoe Co. v. United States (1962), and especially New York v. Kraft Gen Foods, Inc. (1995). In the latter, Judge Kimba Wood held that *"Cross-price elasticity is a more useful tool than own-price elasticity in defining a relevant antitrust market. Cross-price elasticity estimates tell one where the lost sales will go when the price is raised, while own-price elasticity estimates simply tell one that a price increase would cause a decline in volume."* Werden explained Judge Wood put too much weight on that variable, although the court did not mention the size of these elasticities, concerned only with the statistical significance of these measures. Werden criticised the analysis stating that *"It is most unfortunate that the case law has focused on cross elasticities of demand in the delineation of markets. This focus obscures the essential link between market delineation and the underlying market power inquiry."* By the time the paper was written, Werden's perspective did not consider using the aggregate diversion ratio as Shapiro suggested.




**Source:** CADE

- $DR_{AB}$ = 70 or 70%
- $DR_{AC}$ = 30 or 30%

The DR can be used to identify and categorise the closer substitutes; hence, it informs us about which firms exert the more competitive pressure over the other. A product/company that has a 70% DR is a close substitute. It makes more sense to include close substitutes (i.e. with a high DR) in the relevant market, although this variable is not the only one used in delimiting a market.

Several different methodologies can be used to calculate the diversion ratio of companies. For example, one can employ studies about patterns of consumer switching. Additionally, one can analyse how consumers reacted to price changes or product shortages in the past. This data can be collected through simple econometric methods in the scope of an event study[24] and even directly from consumers, with opinion polls. From a quantitative/conceptual perspective, the diversion ratio may be measured in terms of the own-price elasticity or the cross-price elasticity, as seen below:

**Equation 5 - Unit Diversion Ratio**

$$D_{ij} = \frac{\boldsymbol{\varepsilon_{ij} q_j}}{\boldsymbol{\varepsilon_{ii} q_i}}$$

= own-price elasticity (in absolute value)
= cross-price elasticity (in absolute value)
= quantity of i
$q_j$= quantity of j

[24] In the INEOS/Kerling case, to assess the relevant geographic market of chemicals, the Directorate-General for Competition monitored the demand behaviour of competitors when one of the INEOS plants in England was shut down due to an accident. The European authority calculated the diversion ratio between suppliers when the factories halted. It is noteworthy that the decrease in production was completely atypical and not related to the company's strategic decisions—an ideal scenario for calculating the diversion ratio.

Suppose the brands A and B sell the same amount of goods. And suppose A has an own-price elasticity ( ) of -2 and a cross-price elasticity with B ( ) of +0.5. Then, the diversion ratio from A to B will be $\frac{0.5}{2}$, i.e. 0.25 (25%). Hence, one-fourth of the units A loses switch to B.

There are other ways of measuring the proximity between products/competitors or the likeability that competitors will capture each other's sales:

**Table 4 - Measures for ranking substitutes**

| NAME | FORMULA |
|---|---|
| Cross elasticity of demand | $\varepsilon_{ij}$ |
| Unit diversion | $\varepsilon_{ij}q_j$ |
| Sales diversion | $\varepsilon_{ij}p_jq_j$ |
| Unit diversion ratio | $\frac{\varepsilon_{ij}q_j}{\varepsilon_{ii}q_i}$ |
| Diversion ratio by currency (real, dollar, etc.) | $\frac{\varepsilon_{ij}p_jq_j}{\varepsilon_{ii}p_iq_i}$ |
| Relative unit diversion ratio | $\frac{\varepsilon_{ij}s_iq_j}{\varepsilon_{ii}s_jq_i}$ |

**Source:** Adapted from WERDEN, Gregory. Demand elasticities in antitrust analysis, 1998, p. 405.

= own-price elasticity of i (in absolute value)

$\varepsilon_{ij}$= cross-price elasticity between i and j (in absolute value)

$p_i$= price of i

$q_i$= quantity of i

$s_i$= share of i

$p_j$= price of j

$q_j$= quantity of j

$s_j$= share of j

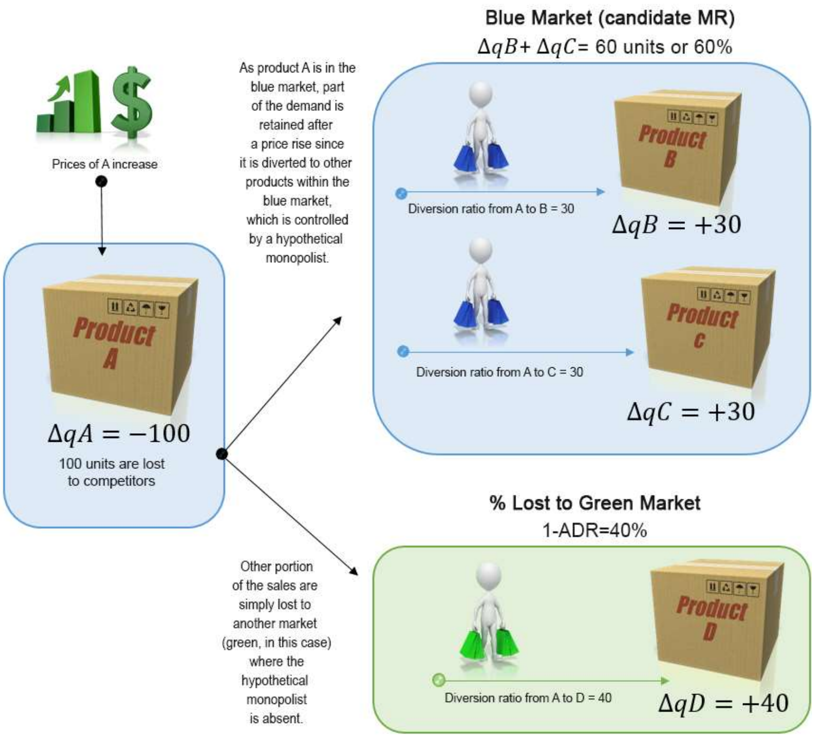
Having analysed the concept of diversion ratio, we move on to the concept of aggregate diversion ratio.

### ii. Notions of Aggregate Diversion Ratio

The aggregate diversion ratio (ADR) is the fraction of overall sales lost by a player's product in one side of the market to other products of this same player in the candidate market.

In the example below, suppose the candidate market is the blue one, formed by products A, B, and C. Product D, on the other hand, is in a different but adjacent market (the green market), and is a partial substitute for A, B, and C.

**Figure 7 - Aggregate Diversion Ratio (ADR) for the blue hypothetical market**




- $DR_{AB}$ = 30 or 30%
- $DR_{AC}$ = 30 or 30%
- $DR_{AD}$ = 40 or 40%
- $ADR_{A\text{->}Blue}$ = 30 + 30 = 60 or 60%

In the example above, the aggregate diversion ratio of A to the other members of the blue market—i.e. B and C ($ADR_{A\text{->}BC}$)—is given by the sum of the individual diversion ratios of A to B ($DR_{AB}$) and A to C ($DR_{AC}$).

Hence, in doing a SSNIP for a single product, the hypothetical monopolist

(i) does an SSNIP in product A, increasing its price;

(ii) effectively loses 1-ADR of A's demand;

(iii) but recovers part of A's lost demand, which is diverted to B and C, which constitutes the ADR.

Based on this value one can analyse a relevant market, as seen below.

### iii. The ADR applied to HMT

To simplify the formulas, the ADR will be represented by the letter A in the analyses below.

A straightforward measure of a hypothetical monopolist's demand loss could be given by:

**Equation 6 - Estimate loss after an SSNIP, considering a linear demand**

$$P = (1 - A)\, s\, \varepsilon$$

s = SSNIP
1 - A = Demand lost to an adjacent market (or consumers that stopped buying)
$\varepsilon$ = Own-price elasticity of demand
$P$ =sales lost due to s
**(Farrell & Shapiro, 2008, p. A1)**

Unlike with the critical loss analysis, we assume that all market players are effectively at their point of maximum profit, i.e. they set prices according to the Lerner formula, pre-merger, which implies the profit margin is the inverse of the own-price elasticity of demand ( $= 1/$ ), assuming it is a heterogeneous Bertrand market.

Thus, using the Lerner index, the formula above can be modified as follows:

**Equation 7 - Estimated loss**

$$P = (1 - A)\frac{s}{m}$$

For a profitable price rise, the incurred loss (that is, the loss that was not captured by the aggregate diversion ratio) cannot be greater than the critical loss. Hence, it should be less than or equal to the critical loss, as seen below:

**Equation 8 - Tradeoff of the hypothetical monopolist (properly defined relevant market)**

$$(1 - A)\frac{s}{m} \leq \frac{s}{(s + m)}$$

Simplifying[25] the calculation:

**Equation 9 - HMT, linear break-even analysis[26] according to the ADR methodology**

---

[25] To get to this formula, we can simplify the equation following these steps:

| | | | |
|---|---|---|---|
| *((1-A)*s/m)*((s+m)m) | =< | (s/(s+m)) * ((s+m)m) | (multiplying both sides by ((s+m)m) |
| *((1-A)*s/m)*((s+m)m) | =< | (s/(s+m))* ((s+m)m) | (cancel the m from the right side and the (s+m) from the left side) |
| * (s+m)s(1-A) | =< | sm | (simplified result) |
| * (s+m)(1-A) | =< | m | (cancelling out the “s” from both sides) |
| * s-sA+m-mA | =< | m | (multiplying the left-hand side) |
| * s+m-A(s+m) | =< | m | (rearranging the terms of the equation with A in the left-hand side) |
| * s+m-m | =< | (s+m)A | (inverting the sides of the terms in bold) |
| * s | =< | (s+m)A | (simplifying) |
| * s/(s+m) | =< | A | Final result |

[26] Hence, if each company has a single product and sets its price to maximise profits, considering the price of everyone else, and if the demand for each product is linear in price changes from the pre-merger price, then a symmetric group of products forms a relevant market under break-even analysis if and only if the condition above is observed (i.e. if the ADR is greater than the critical loss).

$$A \geq \frac{s}{(s + m)} \rightarrow \text{well} - \text{defined market, with a profitable SSNIP}$$

$$A < \frac{s}{(s + m)} \rightarrow \text{ poorly defined market, without a profitable SSNIP}$$

(Farrell & Shapiro, 2008, p. 5)
s = SSNIP
A = aggregate diversion ratio
m = price-cost margin

Hence, if the ADR is greater than or equal to the critical loss, the market will be properly delimited. If not, the relevant market needs to be expanded. This is the version of the profit-maximising test:

**Equation 10 - HMT, maximum profits (linear)**

$$A \geq \frac{2s}{(m + 2s)}$$

Source: Farrell & Shapiro (2008, p. 15)

We stress companies may not price according to the Lerner index or optimise their profits in the pre-merger period. However, the exercise starts from this assumption (that profits are maximised and companies follow the Lerner index) because it uses "pre-merger evidence" to base its calculations, presuming market players behave rationally.

Additionally, unlike the traditional critical loss test, which assesses the SSNIP of all market products at the same time, the ADR uses the SSNIP sequentially, requiring that only a single product in the market meet the SSNIP condition for the relevant market to be properly delimited.

Furthermore, it is ideal that these tests (critical loss and ADR), however different in concept, are analysed in a concurrent and complementary manner so as to inform the authority about merger and pre-merger evidence together.

### iv. ADR and Cost Pass-Through Rate

Farrel & Shapiro (Recapture, Pass-Through, and Market Definition, 2010) introduced a new market delimitation methodology, combining the ADR with the cost pass-through rate (CPTR).

According to the authors:

> This method reinterprets the difference in pricing incentives between the HM and premerger firms from the cost side. We discuss this interpretation in terms of incentives to cut price rather than incentives to raise price; the substantive conclusions are unaffected. Prior to the merger, when Firm A cuts Product 1's price and sells additional units, some of these sales displace (or cannibalize) profitable sales of substitute products,

reducing the profits of those products' owners. Economists call this a "pecuniary negative externality." When these substitutes are added insider substitutes, the HM accounts for, or "internalizes," this negative externality.

To illustrate, consider a merger between Firms A and B, which sell the only two high-end printers, Printers 1 and 2 respectively. If high-end printers are a relevant market, this is a merger to monopoly; but the merging firms argue that high-end printers are too narrow a range of products and will fail the HM test, so that mid-range printers should be included in the relevant market. Doing so lowers the measured shares of Printers 1 and 2, and would thus make some less worried about the merger, although it obviously does not change the merger's true effect. Suppose that before the merger Printers 1 and 2 each sell for $100,000, and each printer sold generates a profit margin, or contribution towards fixed costs, of $40,000 **[M = 40%]**. This is one of the three key pieces of data required by our method. For the second key piece of data, suppose the diversion ratio between the two high-end printers is 25 percent, meaning that 25 percent **[A = 25%]** of the sales captured by cutting the price of one printer come at the expense of the other printer. For a candidate market such as this one that contains only two products, the diversion ratio is the same as the group recapture ratio [or TAD]. In pricing Printer 1, how do the incentives of a HM controlling both printers differ from those of Firm A? Since the diversion ratio is 25 percent, for every four extra units sold of Printer 1 when its price falls, the HM sells one fewer unit of Printer 2, which had generated a profit margin of $40,000. Dividing this by the four extra units of Printer 1, the HM loses $10,000 of profits on Printer 2 for each extra unit sold of Printer 1. This $10,000 amounts to a marginal cost of Printer 1 that the HM faces but that Firm A did not. This opportunity cost affects pricing incentives for Printer 1, just as would any other marginal cost. Higher marginal costs generally lead to higher prices. But how much higher? Suppose Firm A passes through higher marginal costs for Printer 1 to higher prices at a 60 percent rate **[CPTR = 60%]**. This is the third key piece of data for our method. Then the $10,000 extra marginal cost will lead to a price increase for Printer 1 of $6000, which equals 6 percent of the prevailing price of Printer 1. **[M * A * CPTR = 40% * 25% * 60% = 6% > 5% SSNIP]** But this answers precisely the question posed by the HM test: would the HM raise price by at least a SSNIP, usually taken to be 5 percent of the prevailing price? *(FARRELL & SHAPIRO, Recapture, Pass-Through, and Market Definition, 2010)*

Thus, we seek to assess if there are incentives for the hypothetical monopolist to cut prices or not in case it controls the relevant market. In doing this analysis, the authors claim the relevant market is properly defined in the following case:

**11 11 - Proper relevant market in a symmetrical case**

$$\mathrm{CPTR} * \mathrm{A} * \mathrm{m} \geq \mathrm{s}$$

$\mathrm{CPTR_N} = \mathrm{Cost\ pass-through\ rate}$
$\mathrm{A_N} = \mathrm{Aggregate\ diversion\ ratio}$
$\mathrm{m} = \mathrm{Profit\ margin}$
$\mathrm{s} = \mathrm{SSNIP}$
**Source:** (Farrell & Shapiro, Recapture, Pass-Through, and Market Definition, 2010, p. 592)

The authors recognise the two rules do not cover all possible situations; some cases fall into a sort of "grey zone".

For competitive markets, the CPTR is given by

**Equation 12 - CPTR of a group of goods in competitive markets with linear supply**

$$CPTR_{N-Competit} = \frac{\kappa\, \epsilon^{supply}}{(\epsilon^{supply} + \epsilon^{demand})}$$

$\epsilon^{supply} = Elasticity\ of\ supply$
$\epsilon^{demand} = Elasticity\ of\ demand$
$\kappa = Share\ of\ the\ analysed\ group\ (in\ virtue\ of\ an\ external\ good)$
**Source:** (Farrell & Shapiro, Recapture, Pass-Through, and Market Definition, 2010, p. 596)

**Equation 13 - CPTR of a group of goods in a Cournot homogenous model with linear demand**

$$CPTR_{N-Cournot} = \frac{G}{(N + 1)}$$

$G = Companies\ in\ the\ group$
$N = Companies\ in\ the\ general\ market\ (including\ external\ or\ adjacent\ goods)$
**Source:** (Farrell & Shapiro, Recapture, Pass-Through, and Market Definition, 2010, p. 596)

**Equation 14 - CPTR in an oligopoly of differentiated products**

$$CPTR_{N-Differentiate} = \frac{\epsilon}{\epsilon - 1 + \frac{p}{\epsilon} * \frac{d\epsilon}{dp}}$$

$\epsilon = Own-price\ elasticity\ of\ demand$
$p = price$
**Source:** (Farrell & Shapiro, Recapture, Pass-Through, and Market Definition, 2010, p. 596)

These are examples of how to use the ADR to have a broader perspective of pre-merger pricing information. A comprehensive study by the RBB (2014) shows how to measure and analyse the CPTR in the scope of antitrust law.

### v. Other cases

There are other ways of implementing the ADR methodology, as described in Annex I of this document. That is, the ADR test changes if the following factors are taken into account:

- if the demand is residual;

- if the direction of prices matter;

- if there are spillover effects in the market;

- if it is a multi-product firm;

- if it is intended to implement a hypothetical cartel test in multi-product firms;

- if the ADR is used as a ceiling for critical loss;

- if the ADR is applied to a single product;

- if the ADR is applied to a situation of asymmetric diversion ratios;

- if there is a retention rate.

All these directly affect market definition and are exemplified in Annex I of this document.

### c. Simulation methodology

For a SSNIP in markets of differentiated products, some authors, such as Davis & Garcés (2010, p.216) mention one can run not only a complete simulation of the merger effects (whose methodology is in their book's eighth chapter) but also a partial simulation. Considering, for instance, a monopolist has two products and raises their prices, the relevant market is well defined (and a SSNIP is possible) if:

**Equation 15 - CPTR in an oligopoly of differentiated products**

$$\frac{p_1 - c_1}{p_1} \leq \frac{1}{\eta_{11}} + \frac{p_2 - c_2}{p_1} DR_{12}$$

$$\frac{p_2 - c_2}{p_2} \leq \frac{1}{\eta_{22}} + \frac{p_2 - c_2}{p_1} DR_{21}$$

$p_1$= price of product 1 * E
$c_1$ = cost of product 1
$p_2$ = price of product 2 * E
$c_2$ = cost of product 2
SSNIP = 5%, as a rule
E = 1 + SSNIP
$\eta_{11}$= Own-price elasticity of product 1
$DR_{12}$ = Diversion ratio from 1 to 2
$DR_{21}$ = Diversion ratio from 2 to 1

We emphasise these same data allow for adopting the upward pricing pressure (UPP) methodology, described by Shapiro & Farrell (2010), who delimit the relevant market and offer some insights on the merger effects, especially when calibrated with the proper diversion ratios.

In defining a relevant market, some use a simulation in which all players of a market merge[27]. This happens, for example, in the Full Equilibrium Relevant Market (FERM) test, as mentioned by Ivaldi

[27] See Fiuza (2010).

& Lörincz (2005). The authors  alleged their FERM model diverge from the traditional HMT one since it introduces the notion of equilibrium.

As Davis & Garcés (2010) have noted:

> The idea is based on the observation that the SSNIP test is not an equilibrium test in the sense that it does not compare two situations in equilibrium and therefore it does not compare two situations that would actually be found in the real world. To see why, note that the SSNIP test supposes that a monopolist of a candidate market considers the profitability of a unilateral price increase *assuming no reaction to the price increase by producers of goods outside the candidate market.* In contrast, the FERM allows the goods outside the candidate market to respond by changing their prices so we move to a new "equilibrium" set of prices for all products being sold, but where prices inside the candidate market are set by the hypothetical monopolist. Under FERM there will be a tendency to get narrower markets than under SSNIP because price increases by the hypothetical monopolist will generally be followed by price increases of substitutes outside the candidate market. These in turn will tend to reinforce the profitability of the initial price increase and hence push us toward narrower market definitions.
>
> Notice that the question of whether to hold fixed competitive variables, such as price or quantity of those goods which are outside the candidate market, is related to the question of whether to account for supply substitution in market definition. When considering the constraint imposed by supply substitution parties often argue that expansion of output by firms outside the candidate market will defeat an attempted price increase. Parties argue that the implication is that the market definition should be expanded to include other products. In contrast, in a pricing game reactions by firms outside the market will tend to reinforce price increases by the hypothetical monopolist because firms tend to react to price increases by increasing their own prices, i.e., by restricting their supply. [...] The mechanics of applying the test are identical to the tools used in merger simulation.

That is a fascinating discussion since, theoretically, as Davis and Garcés mention, supply substitution is employed as a way to expand the relevant market. Nonetheless, as seen above, in the context of the FERM test, using "supply substitution" or considering the reaction of players outside the market can in fact create narrower market definitions. This may be due to the particularities of the simulation.

This document does not intend to explain how to run a simulation model. To this end, we recommend reading chapter eight of the book by Davis & Garcés (2010). To understand how to make use of this procedure, we also recommend specific packages, such as Michael Sandfort and Charles

Taragin's package written in the R programming language[28]; a software by Jan Boon , written in Phyton[29]; another one in Matlab by Mario Forni, adapted by Marcelo Ranieri[30]; amongst others. Each model has its own specificities.

On the other hand, one can question why choosing such a complex simulation where every player in the market merge (or even outside the market, in FERM analyses) only to define the relevant market if simply carrying out a UPP simulation between the merging parties would serve, or even running broader test. That is, if there is enough information to simulate a broader merger, delimiting the borders of the relevant market is not so important, as Kaplow has stated.

We should also stress that, as stated in footnote 4 of the US 2010 Horizontal Merger Guidelines, the FTC and DOJ do not adopt the HMT test for every situation—especially if companies in a market are also part of an adjacent market. Rather, for these cases they use the concept of a **hypothetical profit-maximizing cartel of firms that sell products in the candidate and adjacent markets** (with all the products and services involved). Hence, the simulations must adapt to this premise.

### d. Methodology for counterfactual analyses

Counterfactual analysis can be useful in discussing market definition. In this regard, Rubinfeld (2010, pp. 71-72) observes:

> When undertaking a unilateral-effects analysis of a merger, the most desirable source of useful information comes from natural experiments that have arisen in the past. These might involve changes in relative prices, changes in capacity, or changes in the number of competitors in the industry (through entry and exit). While this approach has been used frequently by the competition agencies, the number of litigated cases remains relatively small. Among the more interesting litigated cases are *State of New York v. Kraft General Foods* (the merger of two ready-to-eat cereal companies), *FTC v. Staples Depot* (the proposed merger of two office superstore chains), and *FTC v. Whole Foods* (the proposed merger of two high-end supermarket chains). In each of these cases one of the central questions related to market definition. The question at issue was whether a particular product "category" can appropriately be characterized as a separate relevant market. The plaintiffs' proposed market in *Kraft* was "adult cereals", in *Staples "*consumable office products supplied by office superstores", and in *Whole Foods* it was

[28] See https://cran.r-project.org/src/contrib/Archive/antitrust/, accessed 15 May 2020.

[29] See http://janboone.github.io/competition_policy_and_regulation/Mergers/Mergers.html, accessed 15 May 2020.

[30] See https://bibliotecadigital.fgv.br/dspace/bitstream/handle/10438/1816/MarceloRanieriCardoso2006.pdf, accessed 15 May 2020.

"premium, natural, and organic supermarkets". The key issue in each case was whether retailers in each of the proposed markets were more constrained by *intra-category* competition than by *inter-category* competition. In each of these cases, further issues were raised as to the likelihood that there would be unilateral anticompetitive effects. *Staples* offers a useful example. The Federal Trade Commission provided evidence that prices in two-office-superstore cities were seven percent lower than in one-office-superstore cities. The evidence came in the form of simple comparisons of means and in the form of a more sophisticated regression analysis that attempted to control for factors not-related to price changes. While the regression analysis attempted to replicate a natural experiment, the reality was a quasi-experiment in which a number of factors unrelated to changes in price were varying across cities and over time. It is natural to ask: What is the best experimental design for such a quasi-experiment? One promising answer lies in difference-in-differences analysis (DID). As it would be applied in *Staples*, the basic DID design divides the subjects of study (e.g., firms) into a "treatment group" and a "control group". The treatment (e.g., a reduction in the number of competitors) is applied to the treatment group (e.g., cities in which the merger reduces the number of competitors) and the average outcome (e.g., the resulting average price) associated with the treatment control is compared to the control group (the average prices in cities with no change in the number of competitors). If the outcome in the treatment group is different, we conclude that the treatment has had a causal effect on the outcome, e.g., a reduction in the number of superstore competitors has raised prices of office products in superstores.

Thus, counterfactual analyses may not only improve the debate about the direct and predictable effects of a merger but also help in deciding whether an inter- or intra-category competition is best to cover the relevant competition forces.

As seen above, the counterfactual analysis uses techniques that apply the concepts of "control group" and "treatment group". In the medical field, for instance, these concepts can be used in research to analyse the efficiency of a medicine, in which a treatment group consists of patients who are given the drug and a control group of people who are given a placebo only.

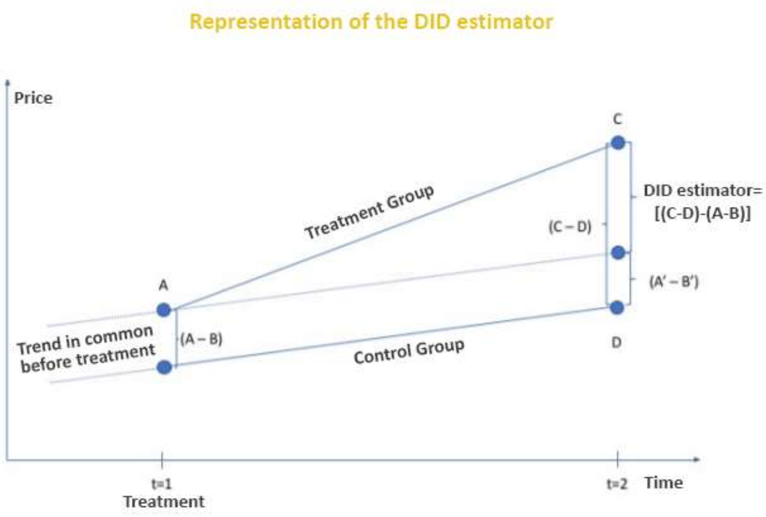
With homogeneous patients and randomly split groups, the difference in patients' health can be attributed to the treatment (in this case, the drug). From a statistical perspective, this difference is captured, for example, by a difference-in-differences technique.

In social terms, however, it is impossible to split the patients into two groups at random. However, once can conduct quasi-experiments. That is, consider the control group as the markets where the concentration ratio remained unaltered, and the treatment group as those where the concentration ratio did change, preferentially for an exogenous reason (e.g. an unexpected accident that caused factory shutdowns). We must ensure that, in both groups, market aspects are similar or at least controllable.

It is also possible, to some extent, to assess how price varies from one place to the other depending on the number of competitors in a market and their entry and exit. Likewise, we can see if the plaintiffs only compete in some places but not in others and, with this information, measure their rivalry by comparing the prices of regions where there is competition with those where there is not.

The DEE has already tried to assess the impact of an antitrust intervention by deeming the intervention as the treatment and similar markets where no intervention took place as the control group:

**Figure 8 - Difference-in-differences estimator**




**Source of the chart:** DEE Working paper no. 4/2019: Mensurando os benefícios de combate a cartéis: o caso do cartel de combustíveis no Distrito Federal[31]

Although we can directly analyse the effects of market concentration, this type of methodology, as mentioned above, can also be used to indicate the most appropriate market delimitation.

For instance, on 17 July 2007, the British firm Ineos Group Limited ("Ineos") notified the European Commission about its acquisition of the Norwegian company Kerling ASA ("Kerling"). Both companies produced PVC. There were some doubts regarding the geographic dimension of the relevant market. This is because the two parties were the only domestic manufacturers of PVC in the United Kingdom. Together, they accounted for 66% of the British market in 2006 (based on volume),

[31] Available at http://www.cade.gov.br/acesso-a-informacao/publicacoes-institucionais/publicacoes-dee/Documentodetrabalho042019Mensurandoosbenefciosdecombateacartis_ocasodocarteldecombustveisnoDistritoFederal.pdf , accessed 15 May 2020.

whilst the rest of the PVC in the market was supplied by imports, chiefly from suppliers located in mainland Europe.

Thus, the Directorate-General for Competition (DG COMP) had to decide whether the UK could be considered an antitrust market for PVC or, alternatively, if the market should be defined in a broader manner as “Western Europe”.

To define the relevant geographic market of chemical products for this merger, the European authority noted that Ineos’ factory in Barry, UK, had to unexpectedly stop its operations in June 2004. Due to a close connection between the production of its plants, the interruption affected the output of Ineos factories in Runcorn, UK. As a result, the volume of PVC Ineos manufactured in the UK was substantially affected (with a 10% loss in the average annual sales of 2002–2006). For a five-month period, the plant manufactured circa 50% of its maximum technical capacity (compared to 90% under normal conditions). This interruption allowed the authority to see how international competitors in the UK reacted to a sudden drop in PVC supply.

The assessment revealed Kerling did not capture in whole the volumes Ineos lost since customers resorted to imports as a supply alternative.  Moreover, Kerling’s profit margins did not increase in the period. This indicates a supposed monopolist in the British geographic market could have its price increases constrained by imports.

Similarly, the Brazilian Braskem/Solvay case merger(2014) involved the PVC market and raised doubts about the geographic dimension of the market. There was also comparable situation where an exogenous factor made some factories unexpectedly interrupt their activities for some time. Consequently, the DEE could make a test with the difference-in-differences model, using as the control group the price behaviour of other resins whose production was not interrupted. The conclusion, however, differed from that of DG COMP. In Brazil, the shutdown was responsible for increasing domestic prices but did not increase importing nor did the imports maintain the price level.  For this reason, CADE understood the Brazilian PVC market had a national dimension.

### e. Notes on quantitative methodologies

We did not intend to exhaust the debate about quantitative models used to define a market. Rather, our intention was to present a few recognised methodologies employed in defining a relevant market. In the annexes of this document, we discuss other matters and delve into the topics of aggregate diversion ratio and critical loss, showing how these methods are relevant and complex to understand at the same time.

## 3. CADE'S CASE LAW

In a previous work, we analysed CADE's precedents (Castro, 2017, p. 205) with the purpose of assessing how permeable the case law is to the quantitative discussion above.

To this end, we employed an empirical and inductive strategy, looking at 28 technical terms and their frequency in the authority's decisions. This methodology was also adopted by other scientific works, such as Cronin (1984) and Feist (1997), and is mentioned as a recommendation by Posner (2000).

Against this backdrop, we selected 6,732 documents with votes issued by the commissioners of CADE from 2004 to 2014. We stress a vote is not the same as a proceeding. Some may have been counted doubly as several commissioners may vote in one same proceeding. It is also possible that a case is brought before the commissioners more than once due to postponements or because commissioners request access to the case records for analysis. Every time a case was considered by the commissioners in a hearing, we counted it as a precedent. We also did not evaluate the final version of the votes but only the preliminary version commissioners bring before their equals when voting.  That is, this count does not calculate the exact number of precedents but is a proxy used to measure the importance of quantitative discussions in CADE's decision-making process.

See below the checked terms and their occurrence in the sample (Table 5).

**Table 5 - Terms, number of occurrences, and frequency**

| Term | Number of occurrences | Frequency in the sample (%) |
|---|---|---|
| hypothetical monopolist test | 50 | 0.74% |
| Upward pricing pressure | 3 | 0.05% |
| critical value of cost reduction | 0 | 0.00% |
| critical elasticity | 5 | 0.07% |
| critical value | 4 | 0.06% |
| critical loss | 7 | 0.10% |
| difference in differences | 0 | 0.00% |
| differences-in-differences | 0 | 0.00% |
| diff-in-diff | 0 | 0.00% |
| aggregate diversion ratio; | 0 | 0.00% |
| diversion ratio | 0 | 0.00% |
| Cross elasticity of demand | 26 | 0.39% |
| break-even | 2 | 0.03% |
| Almost Ideal Demand System | 1 | 0.02% |
| Antitrust logit model | 1 | 0.02% |
| PCAIDS | 1 | 0.02% |
| PC-AIDS | 0 | 0.00% |
| Cointegration | 12 | 0.18% |
| Logit | 5 | 0.07% |

| | | |
|---|---|---|
| **Probit** | 0 | 0.00% |
| **p-value** | 4 | 0.06% |
| **hypothesis testing** | 7 | 0.10% |
| **dummy** | 5 | 0.07% |
| **HHI** | 519 | 7.71% |
| **Relevant market** | 2930 | 43.52% |
| **market share** | 532 | 7.90% |
| **profit** | 617 | 9.17% |
| **cost** | 1807 | 26.84% |

**Source:** Castro (2017, p. 205)

Frequent terms such as "relevant market" (cited in 43.52% of the sample) and the averagely cited "HHI" (7.7% of the sample) contrast with less frequent terms, such as the marginally mentioned "critical loss", "upp", and "diversion ratio" (less than 0.2% of the sample) and "difference in differences", which had no single mention.

It is true, however, that for simpler cases CADE opts for using specific filters, saving resources for the more potentially harmful cases. This may explain why specific or quantitative terms had only a few mentions in the authority's decisions.[32]

If CADE uses some sort of selection or filter to save resources, based on simplified rules and especially market share, this may concentrate too much of the authority's methodologic expertise in some departments (such as the DEE). Thus, it is important that these techniques are made known.

CADE has adopted the HMT as a critical loss or critical elasticity test in only a few cases (Table 6).

After 2010, when the DEE launched its study on market definition, the authority started using fewer tests of price co-movement compared to critical loss tests. In some cases, such as the Braskem/Solvay merger (2014), CADE employed price co-movement tests to criticise the employment of the methodology.

We could not find a case in which CADE simulated the merger of all players in a market, as in the FERM style. Moreover, the cost pass-through rate is not yet an often-used variable to define the relevant market in competition analyses.

**Table 6 - Cases involving a quantitative analysis of the relevant market**

[32] Another possible reason is that CADE does not employ an in-depth empirical analysis (in the votes, at least).

| Case | Price co-movement | Critical Loss | Aggregate Diversion Ratio | Counterfactual Analysis |
|---|---|---|---|---|
| (CVRD/Ferteco/Socimex/Caemi/Samitri merger, 2005) | X[33] | | | |
| (Suzano/Ripasa merger, 2007) | X | | | |
| (DM/Hypermarcas merger, 2008) | X[34] | | | |
| (Petrobrás/Ipiranga/Braskem/Quattor merger, 2008) | X[35] | X[36] | | |
| (Owens Corning/ SaintGobain merger, 2008) | X[37] | | | |
| (Braskem/Quattor merger, 2011) | X[38] | X[39] | | |
| (Sadia/Perdigão merger, 2011) | X[40] | X[41] | | |
| (Braskem/Solvay merger, 2014) | X[42] | X[43] | | X[44] |
| (JBS/Massa Leve merger, 2014) | | X[45] | | |
| (Capsugel/Genix merger, 2015) | X[46] | X[47] | | |
| (Tigre/Condor merger, 2015) | | X[48] | | |

[33] Vol. 4 of the case records (doc. no. 0032754), Case no. 08012.000640/2000-09 / p. 1079; vol. 5 of the case records (doc. no. 0032756), Case no. 08012.000640/2000-09, p. 1322; vol. 10 of the case records (doc. no. 0032764), Case no. 08012.000640/2000-09, p. 2673; vol. 11 of the case records (doc. no. 0032765), Case no. 08012.000640/2000-09, p. 2996; vol. 12 of the case records (doc. no. 0032767), Case no. 08012.000640/2000-09, p. 3177; vol. 13 of the case records (doc. no. 0032770), Case no. 08012.000640/2000-09, p. 3343.

[34] Vol. 1 of the case records (doc. no. 0173941), Case no. 08012.009107/2007-71, p. 254 (part of this document was produced by the plaintiffs and is not necessarily part of the vote).

[35] Vol. 12 of the case records (doc. no. 0147449, p. 81 and following).

[36] Vol. 12 of the case records (doc. no. 0147449, p. 81 and following).

[37] Vol. 7 of the case records (doc. no. 0128887, p.35, 43, 168); vol. 8 of the case records (doc. no. 0129026, p.60).

[38] Vol. 8 of the case records (doc. no. 0031283, p. 142).

[39] Vol. 8 of the case records (doc. no. 0031283, p. 76).

[40] Vol. 16 of the case records (doc. no. 0037516, p.54).

[41] Vol. 16 of the case records (doc. no. 0037516, p.54). See Annex 3 of Vote of the Rapporteur Commissioner (doc. no. 0037516, p.414).

[42] Vol. 18 of the case records (doc. no. 0095138, p.294).

[43] Vol. 18 of the case records (doc. no. 0095138, p.311).

[44] Vol. 18 of the case records (doc. no. 0095138, p.323), Technical Opinion no. 24/2014/DEE/CADE.

[45] Technical Opinion no. 12/2014, by the Department of Economic Studies of CADE, Vol. 2 of the case records (doc. no. 0093360, p. 221).

[46] Technical Opinion no. 002/2015, by the Department of Economic Studies of CADE (doc. no. 0037503).

[47] Technical Opinion no. 002/2015, by the Department of Economic Studies of CADE (doc. no. 0037503).

[48] Technical Opinion no. 19/2015, by the Department of Economic Studies of CADE (doc. no. 0053464 and 0056002).

| | | | | |
|---|---|---|---|---|
| (Tam/Iberia/British merger, 2017) | | X[49] | X[50] | |
| (Ultragaz/Liquigás merger, 2018) | | X[51] | | |

**Source: the DEE, based on CADE´s precedents**

## 4. CONCLUSION

The term relevant market can have different meanings. With analytical tools, it seems there is no right or wrong in making such definitions; additionally, there is certainly some room for theoretical disagreement. When applied to real cases, these disagreements may translate into diverse market definitions.

Moreover, theoretical differences may be latent or occult even when quantitative matters are not known or applied. Only when we shed light on quantitative factors does this kind of divergence appears and becomes relevant. Furthermore, we can use different methodologies at the same time in defining a market, as seen in the TAM/Iberia/British and the Braskem/Solvay mergers. This gives us a broader perspective on how to better fix the borders of a relevant market.

There is still much to discuss on relevant markets. For instance, Lapo Filistrucchi (2008) proposed a test to determine the relevant market in two-sided markets and revealed its complexity. Similarly, Daniel Mandrescu (2018) suggests applying the SSNIC or SSNDQ tests over the SSNIP in dealing with online platforms or a market where one side is free:

> Modifying the price oriented SSNIP into a cost oriented test, would mean that the purpose of the test would be to assess whether the concerned undertaking is capable of imposing a small but significant non-transitory increase in cost for customers in a profitable manner (SSNIC). In the context of such a test, the costs for customers in the case of zero-priced markets are divided into information and attention costs. Information costs refer to the amount of data that the customer needs to provide in order to make use of the free product or service. Attention costs refer to the exposure of customers to advertisements during their use of the zero-priced product or service. Both types of costs can be identified in the case of zero-priced products or services offered by online platforms. [...] Testing demand substitutability based on quality entails some similar practical complexities as in the case of a cost-based test despite the soundness of its economic foundation. In the context of a quality-oriented test, the core question will concern the effect of a small but significant non transitory decrease in quality (SSNDQ), which is comparable to an increase of price from an economic perspective.

Naturally, the so-called zero-price markets generally monetise somehow, sometimes on only one of its sides. Hence, one needs a wider approach to incorporate the paid side when analysing a

[49] Technical Opinion no. 35/2016, by the Department of Economic Studies of CADE (doc. no. 0261515)

[50] Technical Opinion no. 35/2016, by the Department of Economic Studies of CADE (doc. no. 0261515)

[51] Technical Opinion no. 29/2017, by the Department of Economic Studies of CADE (doc. no. 0379359, p.3)

market definition test, as Filistrucchi (2008) proposes. Additionally, unlike the SSNIP test, whose methodology is broadly known, we have yet to discuss how to assess and approach the SSNIC and SSNDQ tests.

In its 2016's Guide for Horizontal Merger Review, CADE introduced the possibility of studying whether a hypothetical monopsonist test could be used instead of the traditional SSNIP test. In these cases, the authority has to analyse how much the monopsonist decreases the price in one link of the market to increase it in the subsequent link. Likewise, there is no well-established strategy for doing this.

These are some of the challenges that antitrust authorities face from the methodological perspective in analysing a relevant market. In the same manner, if there is the hypothetical merger of two innovative drugs yet to be released and aimed to cure diseases that are so far incurable, it is possible that a traditional SSNIP test is insufficient for this new or emerging market where potential competitors are being acquired. The SSNIP test of the HMT is made for markets that are already existent and well established. For other challenges, it is important that the competition authority is open to discuss the best way to solve them with society.

These difficulties, in the end, remind us that a relevant market is simply an analysis tool and that Kaplow's criticism also tends to echo these new challenges.

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## 6. ANNEXES

As mentioned, there are different ways to delve into the aggregate diversion ratio and the critical loss. Below, we illustrate it with some discussions on the topic.

### a. Annex I - Other means of the ADR test

#### i. Basic ADR

The text above mentions solely the ADR test, stressing that the relevant market is well-defined if and only if:

$$A \geq \frac{s}{(s + m)}$$

$s = SSNIP$
$A = ADR$
$m = price - cost\ margin$
Source: (Farrell & Shapiro, 2008, p. 5)

In addition to it, we can mention other useful means.

#### ii. ADR with residual demand

According to Farrell & Shapiro (2008, p. 7), one can analyse the aggregate diversion ratio by the residual elasticity, which would result in other players also changing their prices and elasticity in the case of an SSNIP. In such a case, the relevant market is well-defined if and only if:

**Equation 16 - HMT with residual demand**

$$A^* \geq \frac{s}{(s + m)}$$

$s = SSNIP$
$A^* = ADR\ with\ residual\ demand$
$m = price - cost\ margin$

Farrell and Shapiro believe that:

**Equation 17**

$$A^* < A$$

This is the version of the profit-maximising test:

**Equation 18 - HMT, profit maximisation (linear) with residual demand based on the ADR test**

$$\boldsymbol{A^* \geq \frac{2s}{(m + 2s)}}$$

Farrell & Shapiro (2008, p. 15)

### iii. ADR sensitive to price

Scheffman and Simons (2003) criticise Katz and Shapiro (2003) and O'Brien and Wickelgren (2003) as consumers could be more sensitive to a price increase than a price decrease. Thus, Farrell & Shapiro (2008, p. 9) changed test 1 to comprise such observation. They supposed that if a firm raises its price a little, it will lose sales of 1 + k times greater than the achieved sales upon price decrease.

**Equation 19**

$$A^* \geq \frac{\frac{s}{(s + m)} + k}{1 + k}$$

**Source:** Farrell & Shapiro (2008, p. 9)

In the limit, k would have a as the maximum value$k_{max}$, as observed below:

**Equation 20**

$$k \leq k_{max} = \frac{A^* - \frac{s}{(s + m)}}{1 - A^*}$$

Farrell & Shapiro (2008, p. 9)

### iv. ADR with positive spill-over effects for the monopolist

Besides selling the final product, the hypothetical monopolist firm can also sell additional goods or have other advantages by monopolising a market (such as network economy, increased client fidelity, gaining portfolio power and reputational benefits, amongst others).

Hence, in such a case, the hypothetical monopolist maximises not only the profit of the final product but also the profit derived from adjacent markets in which it operates. The profit margin is no longer only the Lerner index since it comprises, as well, the sale benefits of these other markets ("B"). Thus, Farrell and Shapiro (2008) supported the idea of modifying the profit margin; consequently, diminishing the critical value to be measured:

**Equation 21 - Lerner index with spill-over**

$$m^* = \frac{p - c + B}{p}$$

Farrell & Shapiro (2008, pp. 11-12)

**Equation 22 - New margin with spill-over**

$$m^* = m + b$$

Farrell & Shapiro (2008, pp. 11-12)

**Equation 23 - Elucidation of variable B composition**

$$b = \frac{B}{p}$$

Farrell & Shapiro (2008, pp. 11-12)

Hence, the HMT should include such a discussion. A relevant market would then be properly defined if and only if:

**Equation 24 - HMT with spill-over**

$$A^* \geq \frac{s}{(s + m^*)} = \frac{s}{(s + m + b)}$$

$$s = SSNIP$$
$$A^* = ADR\ with\ residual\ demand$$
$$m = price - cost\ margin$$
$$b = \frac{B}{p} = benefit$$

Farrell & Shapiro (2008, p. 12)

### v. ADR - a multiproduct firm with no spill-over

Moresi, Salop, and Woodbury (2008) suggest another approach when the hypothetical monopolist faces competition in a multiproduct scenario.[52]

Instead of diminishing the critical value to deal with the spill-over issue, as suggested by Farrell and Shapiro (2008), the authors included an element to the numerator that raises the critical value:

**Equation 25 - HMT - Multiproduct in adjacent markets [Break-even]**

$$A_{Candidate} \geq \frac{m\, A_{General} + s}{(s + m)}$$

Moresi, Salop, & Woodbury (2008, p. 8)

[52] "Suppose that the three sellers are The Coca-Cola Company, PepsiCo, and Cadbury Schweppes, and the three products are Cola (e.g., Coke, Pepsi, and RC Cola), Lemon-Lime (e.g., Sprite, Sierra Mist, and 7Up), and Orange Flavor (e.g., Fanta, Tropicana Twister, and Sunkist). Assume that all the brands are substitutes to some degree. For example, the three brands sold by Coca-Cola (e.g., Coke, Sprite, and Fanta) are imperfect substitutes for one another, as are the three brands of the other two firms. Consider next a proposed merger between two of these firms. The antitrust authorities might want to determine whether the three flavours (cola, lemon-lime, and orange) constitute separate, distinct markets or whether they jointly constitute a single, broader carbonated soft drink market. To answer this market definition question, the Merger Guidelines dictate the use of the 'hypothetical monopolist SSNIP test'. [...] the hypothetical monopolist is [...] assumed to produce and sell only the relevant product [e.g. cola] and not any other products (e.g., lemon-lime or orange). However, unlike the hypothetical monopolist, the three firms in the real world do produce and sell brands of other products. This fact has important implications for evaluating market definition under the Merger Guidelines' test."

*(MORESI, SALOP, & WOODBURY, 2008, pp. 1-2)*

**Equation 26 - HMT - Multiproduct in adjacent markets [profit maximisation]**

$$A_{Candidate} \geq \frac{m\,A_{General} + 2s}{(m + 2s)}$$

Moresi, Salop, & Woodbury (2008, p. 5)

**Table 7 - Variables of the above equation**

| | |
|---|---|
| $\boldsymbol{A_{Candidate}}$ | ADR - Diversion of sales into products of the hypothetical monopolist (only in the candidate market) due to an SSNIP in a set of products of the candidate market |
| $\boldsymbol{A_{General}}$ | ADR - Diversion of sales of brands that suffered an SSNIP into all the other brands of the hypothetical monopolist (regardless if in a candidate or adjacent market) |
| $\boldsymbol{s}$ | SSNIP |
| $\boldsymbol{m}$ | Margin |

Thus, there is an apparent contradiction between the formulas since, as evident above, one makes it easy to expand the relevant market whilst the other makes it difficult due to products in an adjacent market. We do not aim to further comment or opine on solving the conflict, but only to inform about it.

### vi. ADR and the Hypothetical Cartel Test (HCT) for symmetric firms

Moresi, Salop, and Woodbury (2017) allege that the US 2010 Horizontal Merger Guideline would have considered a different test to the hypothetical monopolist, which would be the “hypothetical cartel”.

In their work, the authors cite the following excerpt from the Guideline:

> If the pricing incentives of the firms supplying the products in the candidate market differ substantially from those of the hypothetical monopolist, for reasons other than the latter’s control over a larger group of substitutes, the Agencies may instead employ the concept of a hypothetical profit-maximizing cartel comprised of the firms (with all their products) that sell the products in the candidate market. This approach is most likely to be appropriate if the merging firms sell products outside the candidate market that significantly affect their pricing incentives for products in the candidate market.

In this regard, Moresi, Salop, and Woodbury (2017, pp. 25-26) developed the following test:

**Equation 27 - HCT - Multiproduct in adjacent markets [Break-even]**

$$A_{Candidate} \geq \frac{m\,(A_{General} - A_{cartel}) + s}{(s + m)}$$

| | |
|---|---|
| $A_{Candidate}$ | ADR - Diversion of sales into products of the hypothetical monopolist (only in the candidate market) due to an SSNIP in a set of products of the candidate market |
| $A_{General}$ | ADR - Diversion of sales of brands that suffered an SSNIP into all the other brands of the hypothetical monopolist (regardless if in a candidate or adjacent market) |
| $A_{Cartel}$ | ADR - Diversion of sales of any product in the candidate market to all products sold by the hypothetical cartel outside the candidate market |
| $s$ | SSNIP |
| $m$ | Margin |

**Source:** Moresi, Salop, & Woodbury (2017, pp. 25-26)

### vii. ADR as a "bound" [and not a separate test]

O´Brien and Wickelgren (2003, p. 18) proposed using the ADR as a bound to critical loss.

To illustrate the argument, the authors refer to a case in which the merging parties alleged a 60% profit margin and a 10% estimated lost sales. In such a hypothesis, the critical loss would be 0.077 (less than the estimated actual loss).

**Equation 28**

$$\frac{0.05}{(0.05 + 0.60)} = 0.077 < 0.1$$

However, the authors doubt the said "actual loss" because the Lerner index would be a maximum of 0.083 in the hypothesis of the merging firms maximising their profits and the cross elasticity of their products being zero, as follows:

**Equation 29**

$$actual\ loss = 0.05\left[\frac{1}{0.6} - \varepsilon^{cross}\right]$$

**Equation 30**

$$actual\ loss = 0.083 - 0.05\varepsilon^{cross}$$

Therefore, the authors consider 8.3% an acceptable limit of critical loss to be employed by the antitrust authority. Superior to that would oppose the theory, meaning an assumption that would prove the alleged 10% loss unreasonable.

Furthermore, O´Brien and Wickelgren argue the "assumption" would not end the use of critical loss since the test could determine if the estimated loss lies within the critical value and its limit (hypothesis of relevant market expansion) or below the critical value (hypothesis of well-defined relevant market).

As they have stated, in addition to the assumption, it would be necessary to estimate the cross elasticity. For the hypothetical case, for the maximum actual loss not to surpass the critical loss value, a cross elasticity of 0.13 would be sufficient.

**Table 8 - Critical diversion ratio and critical cross elasticity – Linear demand**

| | 1% Price Increase | | 5% Price Increase | | 10% Price Increase | |
|---|---|---|---|---|---|---|
| Margin | Critical Cross Elasticity | Critical Diversion Ratio | Critical Cross Elasticity | Critical Diversion Ratio | Critical Cross Elasticity | Critical Diversion Ratio |
| 10% | 0.91 | 9.1% | 3.33 | 33.3% | 5.00 | 50.0% |
| 15% | 0.42 | 6.3% | 1.67 | 25.0% | 2.67 | 40.0% |
| 20% | 0.24 | 4.8% | 1.00 | 20.0% | 1.67 | 33.3% |
| 25% | 0.15 | 3.8% | 0.67 | 16.7% | 1.14 | 28.6% |
| 30% | 0.11 | 3.2% | 0.48 | 14.3% | 0.83 | 25.0% |
| 35% | 0.08 | 2.8% | 0.36 | 12.5% | 0.63 | 22.2% |
| 40% | 0.06 | 2.4% | 0.28 | 11.1% | 0.50 | 20.0% |
| 45% | 0.05 | 2.2% | 0.22 | 10.0% | 0.40 | 18.2% |
| 50% | 0.04 | 2.0% | 0.18 | 9.1% | 0.33 | 16.7% |
| 55% | 0.03 | 1.8% | 0.15 | 8.3% | 0.28 | 15.4% |
| 60% | 0.03 | 1.6% | 0.13 | 7.7% | 0.24 | 14.3% |
| 65% | 0.02 | 1.5% | 0.11 | 7.1% | 0.21 | 13.3% |
| 70% | 0.02 | 1.4% | 0.10 | 6.7% | 0.18 | 12.5% |
| 75% | 0.02 | 1.3% | 0.08 | 6.3% | 0.16 | 11.8% |
| 80% | 0.02 | 1.2% | 0.07 | 5.9% | 0.14 | 11.1% |
| 85% | 0.01 | 1.2% | 0.07 | 5.6% | 0.12 | 10.5% |
| 90% | 0.01 | 1.1% | 0.06 | 5.3% | 0.11 | 10.0% |

**Source:** O´Brien & Wickelgren (2003, p. 18)

As to the constant elasticity demand, the authors calculated the following:

**Equation 31 - Elasticity calculation**

$$q = ap_1^{-\varepsilon^{own}} p_2^{\varepsilon^{cross}}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

**Equation 32 - Arc elasticity calculation**

$$arc\varepsilon^{own} = \frac{a((1+x)p_1)^{\varepsilon^{own}} p_2^{\varepsilon^{cross}} - ap_1^{-\varepsilon^{own}} p_2^{\varepsilon^{cross}}}{x\left(ap_1^{-\varepsilon^{own}} p_2^{\varepsilon^{cross}}\right)}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

**Equation 33 - Simplifying and imposing symmetry**

$$arc\varepsilon^{own} = \frac{1-(1+x)^{-\varepsilon^{own}}}{x} \neq \varepsilon^{own}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

**Equation 34 - Using arc elasticity to estimate the actual loss in terms of point elasticity**

$$actual\ loss = 2 - (1+x)^{-\varepsilon^{own}} - (1+x)^{\varepsilon^{cross}}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

**Equation 35 - Using Lerner instead of own-price elasticity**

$$actual\ loss = 2 - (1 + x)^{-1/m} - (1 + x)^{\varepsilon^{cross}}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

**Equation 36 - Equating the actual loss with the critical loss and solving own-price elasticity**

$$\varepsilon^{cross} < \frac{\ln(2 - (1 + x)^{1/m} - \frac{x}{x + m}}{\ln(1\ +\ x)}$$

**Source:** O´Brien & Wickelgren (2003, p. 19)

Based on the above formula, the authors calculated the following critical values:

**Table 9 - Critical diversion ratio and critical cross elasticity – Constant elasticity**

| | 1% Price Increase | | 5% Price Increase | | 10% Price Increase | |
|---|---|---|---|---|---|---|
| Margin | Critical Cross Elasticity | Critical Diversion Ratio | Critical Cross Elasticity | Critical Diversion Ratio | Critical Cross Elasticity | Critical Diversion Ratio |
| 10% | 0.38 | 3.8% | 1.05 | 10.5% | 1.14 | 11.4% |
| 15% | 0.17 | 2.5% | 0.56 | 8.4% | 0.71 | 10.7% |
| 20% | 0.09 | 1.8% | 0.33 | 6.7% | 0.47 | 9.4% |
| 25% | 0.06 | 1.4% | 0.22 | 5.4% | 0.32 | 8.1% |
| 30% | 0.04 | 1.1% | 0.15 | 4.4% | 0.23 | 6.9% |
| 35% | 0.03 | 0.9% | 0.10 | 3.7% | 0.17 | 5.9% |
| 40% | 0.02 | 0.7% | 0.08 | 3.0% | 0.13 | 5.0% |
| 45% | 0.01 | 0.6% | 0.06 | 2.5% | 0.09 | 4.3% |
| 50% | 0.01 | 0.5% | 0.04 | 2.1% | 0.07 | 3.6% |
| 55% | 0.01 | 0.4% | 0.03 | 1.8% | 0.06 | 3.0% |
| 60% | 0.01 | 0.3% | 0.02 | 1.4% | 0.04 | 2.5% |
| 65% | 0.00 | 0.3% | 0.02 | 1.2% | 0.03 | 2.1% |
| 70% | 0.00 | 0.2% | 0.01 | 0.9% | 0.02 | 1.7% |
| 75% | 0.00 | 0.2% | 0.01 | 0.7% | 0.02 | 1.3% |
| 80% | 0.00 | 0.1% | 0.01 | 0.6% | 0.01 | 1.0% |
| 85% | 0.00 | 0.1% | 0.00 | 0.4% | 0.01 | 0.7% |
| 90% | 0.00 | 0.1% | 0.00 | 0.3% | 0.01 | 0.5% |

**Source:** O´Brien & Wickelgren (2003, p. 20)

### viii. ADR and SSNIP of one product with asymmetry

Katz and Shapiro (2003) claimed the HMT would not determine an immediate price rise of all products in the markets where the hypothetical monopolist operates. On the contrary, the hypothetical monopolist could raise the price of product 1 first, then product 2 and so on:

> "Up to this point, we have followed the letter of the Merger Guidelines in asking whether the hypothetical monopolist would find it most profitable to raise the price of at least one product of the merging parties by some threshold amount above prevailing levels. However, we are aware that the market definition test often employed in practice is slightly different. Specifically, the test often takes the form of asking whether the hypothetical monopolist would find it most profitable to raise the prices of all of the

products in the candidate market at least 5 per cent above prevailing levels. We now turn to an analysis of this slightly different question. To begin this modified inquiry, one first needs to decide what pre-merger gross margin to apply to products controlled by the hypothetical monopolist. In practice, the gross margins of the merging suppliers are typically taken as representative of the industry because the most reliable data on price and cost readily available usually come from the merging parties. In practice, then, the prices and costs of the merging parties serve as the basis for the hypothetical monopolist calculations.

With these standard working assumptions, one can easily extend our previous analysis to ask about the profitability of imposing an SSNIP uniformly on all of the products in the candidate's relevant market.

The trick is to perform this exercise by raising the price of one product after another in sequence until the prices of all products in the candidate market have been raised. We have already shown that raising the price of one product, Product #1, is profitable for the hypothetical monopolist if the aggregate diversion ratio for that product exceeds the critical loss. After the price of Product #1 has been raised, the same logic applies to Product #2, then Product #3, and so on until the prices of all products in the candidate market have been raised. So long as the aggregate diversion ratio for each product in the sequence does not go down sharply when the price of another product is raised by a small amount, the test we described above will apply to every product in the candidate relevant market."

According to Dajlord, Sørgard and Thomassen (2008):

It is not clear from the wording of the US Merger Guidelines whether the SSNIP test requires a relative increase in the price of one, some or all of the products in the candidate market. The ambiguity of the Guidelines has led to different profitability criteria being used in the literature Harris and Simons derived a criterion based on a uniform price increase on all products in the candidate market. Katz and Shapiro introduced a somewhat different test. They consider the effect of raising the price of only one product and derive a market definition criterion based on the aggregate diversion ratio.

Conversely, Dajlord, Sørgard and Thomassen (2008) point out that Katz and Shapiro (2003) made an error in calculating the critical loss for the price rise of a single product, which led to the test being applied as follows:

Equation 37 - ADR Methodology vs. Dalijor, Sorgard and Thomassen

| Authors | L (Loss) | CL (Critical loss) | HMT - well-defined market |
|---|---|---|---|
| (DALJORD, SØRGARD, & THOMASSEN, 2008) | $\frac{s}{m_1}$ | $\frac{s}{s+m_1}(1+\lambda A)$ | $A_{ji} \geq \frac{s}{m}$ [with symmetry] (One product only) |
| (KATZ & SHAPIRO, 2003) | $(1-A)\frac{s}{m}$ | $\frac{s}{s+m}$ | $A \geq \frac{s}{s+m}$ [with symmetry] (One product only) |

This was acknowledged by Farrell and Shapiro (2008, p. 5):

> "Proposition1 (Katz-Shapiro-O'Brien-Wickelgren): If each firm owns a single product and prices to maximize its profits taking as given all other prices, and if demand for each product is linear in price for small price changes from the premerger price, then a symmetric group of products forms a market under break-even analysis if and only if
>
> A ≥ s / (m + s)
>
> (…)
>
> This is the test for whether a uniform SSNIP, imposed on all the (symmetric) products in the candidate market, is more profitable than the status quo price.
>
> A different condition diagnoses whether the hypothetical monopolist would find an SSNIP imposed on just one product more profitable than the status quo. As Øystein Daljord, Lars Sørgard & Øyvind Thomassen note in *The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro*, J. COMPETITION L. & ECON. (forthcoming 2008), this latter condition is that
>
> A ≥ s /m.
>
> Because this latter condition is less easily satisfied than the one in Proposition 1, Proposition 1 accurately follows the Merger Guidelines test in the symmetric case. As Daljord et al. stress, the asymmetric test follows the Merger Guidelines in asymmetric situations where an asymmetric SSNIP is more apt to be profitable than is a symmetric one."

Thus, the work of Daljord, Sørgard and Thomassen (2008) is acknowledged in the literature. For the authors, Katz and Shapiro focused their analysis only on asymmetric cases regarding all market products (in which case the diversion of product i to product j is equal to the one from product j to product i). But that is not necessarily true.

For them, in an asymmetric SSNIP, the critical loss would be calculated as follows:

**Equation 38 - HMT**

$$\pi_1(p^s) - \pi_1(p) + \pi_2(p^s) - \pi_2(p)$$

Daljord, Sørgard and Thomassen (2008, p. 7)

**Equation 39 - Price calculation**

$$p^s = ((1+s)p_1.p_2)$$

Daljord, Sørgard and Thomassen (2008, p. 7)

**Equation 40 - HMT with all variables**

$$\big((1+s)p_1 - c_1\big)q_1(1-CL) - (p_1 - c_1)q_1 + (p_2 - c_2)q_2(1 + s\eta_{21}) - (p_2 - c_2)q_2 = 0$$

Daljord, Sørgard and Thomassen (2008, p. 8)

$\eta_{21} = Cross - price\ elasticity\ of\ demand\ of\ product\ 2\ in\ relation\ to\ price\ 1$
$p_1 = price\ 1$
$p_2 = price\ 2$
$c_1 = marginal\ cost\ 1$
$c_2 = marginal\ cost\ 2$
$q_1 = quantity\ 1$
$q_2 = quantity\ 2$
$s = SSNIP$
$CL = Critical\ loss$

**Equation 41 - Simplified HMT**

$$(1+m_1)R_1(1-CL) - m_1R_1 + s\eta_{21}m_2R_2 = 0$$

Daljord, Sørgard and Thomassen (2008, p. 8)

**Equation 42 - Sales of j (used in the above equation - simplified HMT)**

$$R_j = p_jq_j$$

Daljord, Sørgard and Thomassen (2008, p. 8)

**Equation 43 - Margin j (used in the above equation – simplified HMT)**

$$m_j = \frac{(p_j - c_j)}{p_j}$$

Daljord, Sørgard and Thomassen (2008, p. 8)

**Equation 44 - Critical Loss with modified values**

$$CL = \frac{s}{s + m_1}(1 + \lambda A)$$

Daljord, Sørgard and Thomassen (2008, p. 8)

**Equation 45 - Relative profit (employed in the above equation – Critical Loss)**

$$\lambda = \frac{p_2 - c_2}{p_1 - c_1}$$

Daljord, Sørgard and Thomassen (2008, p. 8)

In this sense, the authors defend that the actual loss is:

**Equation 46 - Actual loss**

$$L = s * \varepsilon_{ii} = \frac{s}{m_1}$$

Thus the actual loss cannot be greater than the critical loss to have a well-defined relevant market. Therefore:

**Equation 47 - HMT L<CL**

$$\frac{s}{m_1} < \frac{s}{s + m_1}(1 + \lambda A)$$

Daljord, Sørgard and Thomassen (2008, p. 8)

**Equation 48 - Multiplying by the GCD**

$$(s + m_1)s < sm_1(1 + \lambda A)$$

**Equation 49 - Multiplying the terms on both sides**

$$ss + sm_1 < sm_1 + sm_1\lambda A$$

**Equation 50 - Simplifying sm1**

$$ss < sm_1\lambda A$$

**Equation 51 - Cancelling out the "s" on both sides**

$$s < m_1\lambda A$$

**Equation 52 - The formula found by the authors**

$$\frac{s}{m_1} < \lambda A$$

Based on these, Daljord, Sørgard and Thomassen (2008, p. 8) claim that Katz & Shapiro made an error, as they understand that if there is symmetry ($\lambda = 1$) that aimed at raising the price of a single product, a relevant market would be well defined if $\boldsymbol{A_{ji}} \geq \frac{s}{m}$, instead of $\boldsymbol{A_{ji}} \geq \frac{s}{s+m}$, as claimed by Katz and Shapiro.

In case of symmetry of the diversion ratios, we would have:

**Equation 53 - Equivalent ADR**

$$A_{ij} = A_{ji} = A$$

**Equation 54 - Equal losses**

$$L_i = L_j = L$$

Hence, the formula used by Katz and Shapiro (2003) applies an unweighted average of ADR of i and j, as follows:

**Equation 55**

$$\frac{A_{ij} + A_{ji}}{2} \geq \frac{s}{s + m}$$

**Source:** Daljord and Sørgard (2010, p. 4)

However, Daljord and Sørgard (2010) present a weighted average of the ADRs in their analysis for uniform SSNIPs (considering as uniform the SSNIP related to all the products of the hypothetical monopolist and not only one or some):

**Equation 56 - Uniform but asymmetric HMT, with a weighted average of ADRs**

$$\theta_i A_{ij} + (\theta_i - 1)A_{ji} \geq \frac{s}{s + m}$$

**Source:** Daljord, Sørgard (2010, p. 5)

**Equation 57 - Proportion of i to j**

$$\theta_i = \frac{q_i}{q_i + q_j}$$

The authors considered the hypothesis that the ADRs are proportional to the quantities sold:

**Equation 58 - The proportion of ADR to quantity**

$$A_{ji} = \frac{A_{ij}\theta_i}{(1 - \theta_i)}$$

**Source:** Daljord and Sørgard (2010, p. 6)

Therefore, Daljord and Sørgard (2010) establish that asymmetric SSNIP results in a more narrow market in comparison with a symmetric SSNIP if:

**Equation 59 – The asymmetric SSNIP is more conservative**

$$\theta_i < \frac{m}{2(s + m)}$$

**Source:** Daljord and Sørgard (2010, p. 7)

For example, if the profit margin (m) is 30% and the SSNIP (s) is 5%, then the critical theta would be 42.8%. That is, if the share of product 1 (whose price is to be increased) does not surpass 42.8% in comparison to product 2, then the market is properly defined.

In this example, the authors demonstrate a situation where the quantity of j is 4 times that of i. Thus, the share of i ($\theta_i$) and j will be 20% and 80%, respectively.

**Equation 60 - Example provided**

$$0.8\,A_{ji} = 0.2A_{ij}\theta_i$$

**Equation 61 - The above equation simplified**

$$4\,A_{ji} = A_{ij}\theta_i$$

**Table 10 Uniform SSNIP and the SSNIP of a product, considering asymmetries according to the example**

| | Uniform SSNIP | SSNIP of one product only |
|---|---|---|
| Formula | $\boldsymbol{\theta_i A_{ij} + (\theta_i - 1)A_{ji} \geq \frac{s}{s\,+\,m}}$ | $\boldsymbol{A_{ji} \geq \frac{s}{m}}$ |
| Application | $\mathbf{0.2 * 0.2 + 0.}8\text{*}0.05 \geq \frac{\mathbf{0.05}}{\mathbf{0.05+0.3}}$ | $\mathbf{0.2 \geq \frac{0.05}{0.3}}$ |
| Conclusion | $\mathbf{8\% \geq 14.3\%}$ (False) poorly defined RM | $\mathbf{0.2 \geq 0.167}$ (True) Properly defined RM |

**Source:** Daljord and Sørgard (2010, p. 8)

Langenfeld and Li (2014, p. 2) believe that Daljord, Sørgard and Thomassen tried to correct a mistake by Katz and Shapiro, but also made a mistake in their formula. Thus, the authors suggest a new formula:

**Table 11 - Comparison between formulas that intend to impose the SSNIP on only one product**

| Authors | The formula for calculating the profit of a hypothetical monopolist |
|---|---|
| (DALJORD, SØRGARD, & THOMASSEN, 2008) | $\boldsymbol{\big((1+s)p_1 - c_1\big)(q_1 - q_1 CL) - (p_1 - c_1)q_1 + (p_2 - c_2)(q_2 + q_2 s\eta_{21}) - (p_2 - c_2)q_2 = 0}$ |
| (LANGENFELD & LI, 2001) | $\boldsymbol{(p_1 + \Delta p_1 - c_1)(q_1 - \Delta q_1) + (p_2 - c_2)(q_2 + \Delta q_1 A_{12}) \geq (p_1 - c_1)q_1 + (p_2 - c_2)q_2}$ |

**Source:** (LANGENFELD & LI, Asymmetric price increase in critical loss analysis: a reply to Dalijord, Sorgard and Thomassen, 2014)

$\eta_{21} = Cross - price\ elasticity\ of\ demand\ of\ product\ 2\ in\ relation\ to\ price\ 1$

$A_{12} = \text{Radius of the unit diversion between product 1 and 2} = \frac{\epsilon_{21}\, q_2}{\epsilon_{11}\, q_1}$
$p_1 = \text{price } 1$
$p_2 = \text{price } 2$
$c_1 = \text{marginal cost } 1$
$c_2 = \text{marginal cost } 2$
$q_1 = \text{quantity } 1$
$q_2 = \text{quantity } 2$
$s = SSNIP$
$CL = \text{Critical loss}$

According to Langenfeld and Li (2014), Daljord, Sørgard and Thomassen mixed variables with the same name in the formula, but these variables relate to different matters, as sometimes quantity 1 (that is, "$q_1$") refers to the critical loss and sometimes to the actual loss suffered by the monopolist.

Thus, the authors came up with different formulas for what critical loss means, in situations involving the SSNIP of a single product, as can be observed below:

| | (DALJORD, SØRGARD, & THOMASSEN, The SSNIP test and market definition with aggregate diversion ratio: a reply to Katz and Shapiro, 2008) | (LANGENFELD & LI, 2001) |
|---|---|---|
| Critical loss | $\frac{s}{s+m_1}(1+\lambda A)$ | $\frac{s_1}{s_1+m_1-m_2(\frac{p_2}{p_1})A_{12}}$ |

**Source:** (LANGENFELD & LI, Asymmetric price increase in critical loss analysis: a reply to Dalijord, Sorgard and Thomassen, 2014)

Langenfeld and Li (2014) also defend that their formula is in agreement with the discussion held by Moresi, Salop, and Woodbury (2008).

Daljord, Sørgard and Thomassen, (2014) answered such critique by alleging that the test suggested by Langenfeld and Li is the very same suggested by themselves, under the condition of profit maximisation by Lerner.

**Equation 62**

$$\frac{s_1}{m_1} < \frac{s_1}{s_1+m_1-m_2(\frac{p_2}{p_1})A_{12}}$$

**Equation 63**

$$s_1+m_1-m_2\left(\frac{p_2}{p_1}\right)A_{12} < m_1$$

**Equation 64**

$$\frac{s_1}{m_1} < \frac{m_2}{m_1}\frac{p_2}{p_1}A_{12} = \frac{p_2-c_2}{p_1-c_1}A_{12} = \lambda A_{12}$$

Langefeld and Li (2014) disagree with that assessment.

> "First, in their reply, Daljord, Sørgard, and Thomassen suggest we make a claim that we did not. In their telling, our original critique claims that their final criterion in the third step of critical loss analysis for assessing whether the actual loss is greater than the critical loss under the Lerner condition was incorrect. In place of this mistaken criterion, they say, we offered a criterion of our own. As explained above, however, our earlier article critiques only the critical loss formula that Daljord, Sørgard, and Thomassen derived in the first step of the critical loss analysis.
>
> Second, Daljord, Sørgard, and Thomassen misstate the reason our critical loss expression differs from theirs. According to their reply, the difference emerges because they derived their critical loss expression in the first step of critical loss analysis by imposing the profit-maximizing Lerner condition and we derived our critical loss expression without this assumption. Daljord, Sørgard and Thomassen's characterisation is not accurate. Therefore, the reason that the critical loss expression that Daljord, Sørgard, and Thomassen derived differs from the critical loss expression that we derived is not because we did not impose the profit-maximizing Lerner condition, but rather because of an error in Daljord, Sørgard, and Thomassen's derivation, as we explained in our original critique.
>
> Third, Daljord, Sørgard, and Thomassen state that our approach to market definition focuses on critical loss alone. We never stated that one can define the relevant market by focusing on critical loss alone. As explained above, there are three steps in conducting a critical loss analysis, and no one can conduct a critical loss analysis by focusing on critical loss alone. In the 2001 Antitrust Bulletin article in which we first derived the critical loss expression for an asymmetric price increase, we explained the different steps in critical loss analysis clearly" (…)
>
> Fourth, Daljord, Sørgard, and Thomassen state in their reply that when we reach a market delineation different from theirs, it must be the case that we have imposed an assumption on firms' price-setting behaviour that is inconsistent with the Lerner condition of profit-maximization price setting. We disagree. As we explained in Part II, we conducted the Monte Carlo simulation merely to assess under what conditions the critical loss expression derived by Daljord, Sørgard, and Thomassen is likely to differ in some substantial ways from the one that we derived. We never imposed any assumption that firms are not maximizing their profit in setting their prices. We are among the first authors to draw attention to the implications of the Lerner condition for critical loss analysis. Our 2001 Antitrust Bulletin article says that "high premerger margins of the merging firms imply that the merging firms face a relatively inelastic demand and the actual loss of sales is small if the merged firm raises price."

With no intention of wrapping up the discussion, we have only transcribed the authors' opinions.

### ix. ADR and Retention rate

Scheffman and Simons (2003) had already criticised the ADR methodology, mentioning that the Lerner index could not be used to calculate the actual loss of a hypothetical monopolist. Coate and

Simons (2009, p. 6) indicate how the proposed calculations would be implemented based on the Lerner index:

**Table 12 - Expected loss and critical loss**

| (m) Profit margin | E=1/m Elasticity | EC=(0.05/(0.05+m)) Critical loss | L = E*0,05 Expected loss | CE/L | EIMP=EC/0.05 Implicit elasticity |
|---|---|---|---|---|---|
| 0.1 | 10.00 | 33.3% | 50% | 66.7% | 6.667 |
| 0.2 | 5.00 | 20.0% | 25% | 80.0% | 4.000 |
| 0.3 | 3.33 | 14.3% | 17% | 85.7% | 2.857 |
| 0.4 | 2.50 | 11.1% | 13% | 88.9% | 2.222 |
| 0.5 | 2.00 | 9.1% | 10% | 90.9% | 1.818 |
| 0.6 | 1.67 | 7.7% | 8% | 92.3% | 1.538 |
| 0.7 | 1.43 | 6.7% | 7% | 93.3% | 1.333 |
| 0.8 | 1.25 | 5.9% | 6% | 94.1% | 1.176 |
| 0.9 | 1.11 | 5.3% | 6% | 94.7% | 1.053 |
| 1 | 1.00 | 4.8% | 5% | 95.2% | 0.952 |

**Source:** based on Coate & Simons (2009, p. 6)

As observed in the table, the expected loss always surpasses the critical loss for one evident reason: the monopolist is already in a position of profit maximisation before the SSNIP.

That is why—assuming the ADR methodology—a diversion ratio that diminishes the expected loss value is necessary. However, Coate and Simons (2009) bring into question that, supposing all players of a relevant market raise prices simultaneously, why would customers prefer to switch to a set of products with concurrent price rises instead of products with price stability (outside the relevant market)?

The response of Katz and Shapiro (2003, p. 54) to the issue considers the SSNIP is imposed jointly but, overall, in sequence. In other words, if three firms operate in the market [A, B and C] and one firm outside the market [X], then:

First, A raises prices whilst B and C incorporate the aggregate diversion ratio

Second, B raises prices whilst A and C incorporate the aggregate diversion ratio

Third, C raises prices whilst A and B incorporate the aggregate diversion ratio

However, Coate and Simons (2009) allege that, especially in the third moment, customers of firm C will no longer be interested in diverting to B or A because these firms' products already have a considerable price rise. This situation would alter the cross elasticity for consumers of C, who might even prefer to continue where they are. For this reason, the authors suggest there should be a way to model the increase in customers' lack of interest in the relevant market products as their prices rise in a sequential SSNIP. Thus, a retention rate would be needed to better define how the sequential SSNIP occurs.

The authors, however, are very sceptical about using the Lerner index to infer the loss of the hypothetical monopolist (regardless of measuring the elasticity level). Some authors such as Houthakker and Taylor (2010) have already calculated inelastic demands. As observed, it is impossible to have inelastic demand with the above ADR methodology using the Lerner index.

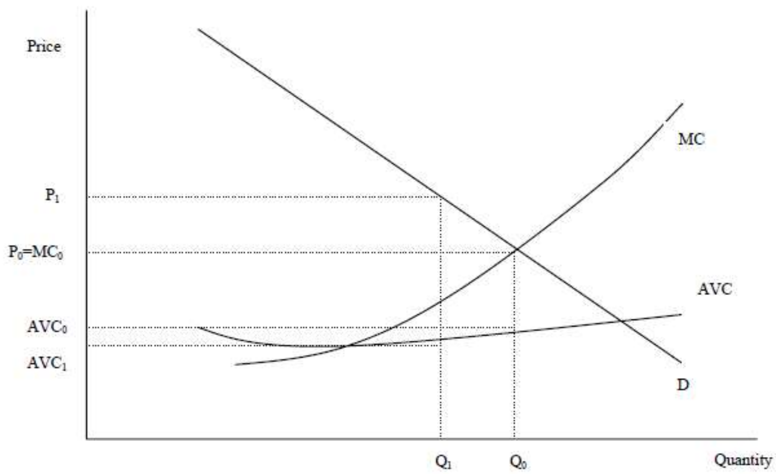
### b. Annex II - Generalised critical loss

#### i. Generalised Critical Loss [Break-even - linear]

Coate and Williams (2005) understand that using AVC as a proxy for the marginal cost poses some problems. Due to this, both authors developed a concept known as generalised critical loss analysis.

To illustrate their point of view, the authors mention the following chart, in which MC means marginal cost, AVC is the average variable cost, $P_0$, $Q_0$ $e$ $AVC_0$ the price, quantity and the CVM before the SSNIP, and $P_1$, $Q_1$ $e$ $AVC_1$ the price, quantity and CVM after the SSNIP.

**Figure 9 - HMT considering a rise of $P_0$ to $P_1$**




**Source:** Coate & Williams (2005, p. 6)

From a break-even perspective, the critical loss analysis would seek the following:

**Equation 65 - Critical loss**

$$P_0Q_0 - \int_0^{Q_0} MC - Fixed\ cost = P_1Q_1 - \int_0^{Q_1} MC - Fixed\ cost =$$

**Source:** Coate & Williams (2005, p. 6)

$P_0$ = initial price

$Q_0$ = initial quantity
$MC$ = Marginal cost
$P_1$ = price after the SSNIP
$Q_1$ = quantity after the SSNIP

**Equation 66 - Quantity after and before the SSNIP**

$$Q_1 = Q_0(1+x)$$

**Source:** Coate & Williams (2005, p. 6)

**Equation 67 - Price after and before the SSNIP (y)**

$$P_1 = P_0\ (1+y)$$

**Source:** Coate & Williams (2005, p. 6)

**Equation 68 - Authors allege rearranging the equations**

$$x = \frac{y}{1+y} + \frac{\int_0^{Q_0} MC - \int_0^{Q_1} MC}{P_0 Q_0 (1+y)}$$

**Source:** Coate & Williams (2005, p. 6)

X = percentage decrease in volume that would make a monopolist indifferent to the Y price rise per current competitive level of P0 to the less competitive level of P1
Y=SSNIP

According to the authors, considering that the average variable cost (AVC) is the area described in the above chart divided by the quantity, the formula could be rewritten as follows in terms of AVC:

**Equation 69 - Critical loss in terms of average variable cost**

$$x = \frac{yP_{0+}\ AVC_0 -\ AVC_1}{P_0 + yP_0 - AVC_1}$$

**Source:** Coate & Williams (2005, p. 6)

The authors also claim accounting data may estimate the contribution margin, considering the average cost before the SSNIP.

**Equation 70 - Contribution margin before the SSNIP**

$$\boldsymbol{MC_{pre} = \frac{P_0 -\ AVC_0}{P_0}}$$

**Source:** Coate & Williams (2005, p. 6)

However, as there is no equilibrium after the SSNIP and for pragmatic reasons, calculating the is not easy$AVC_1$; to simplify, we assume $AVC_0 = AVC_1$. Thus, we have:

**Equation 71 - Rearranged equation supposing a stable average variable cost**

$$x = \frac{yP_0}{P_0 + yP_0 - AVC_0} = \frac{y}{y + MC_{pre}}$$

**Source:** Coate & Williams (2005, p. 6)

Coate and Williams question the validity of the argument according to which $AVC_0 = AVC_1$, as they consider it not to be a realistic assumption. They propose a critical loss structure that allows for a more general cost structure.

**Equation 72 - Critical loss of simplified break-even**

$$x = \frac{y}{1+y} + \frac{\int_{Q_1}^{Q_0} MC}{P_0 Q_0 (1+y)}$$

**Source:** Coate & Williams (2005, p. 9)

**Equation 73 - Average marginal cost**

$$AMC = \frac{\int_{Q_1}^{Q_0} MC}{xQ_0}$$

**Source:** Coate & Williams (2005, p. 10)

**Equation 74 - Rearranged equation inserting the AMC**

$$x\left[1 - \frac{AMC}{P_0(1+y)}\right] = \frac{y}{1+y}$$

**Source:** Coate & Williams (2005, p. 10)

It is possible to define the average marginal contribution margin (AMCM) as follows:

**Equation 75 - AMCM**

$$AMCM = \frac{P_0 AMC}{P_0}$$

**Source:** Coate & Williams (2005, p. 10)

Then, Coate and Williams assumed an accurate measure of critical loss would be:

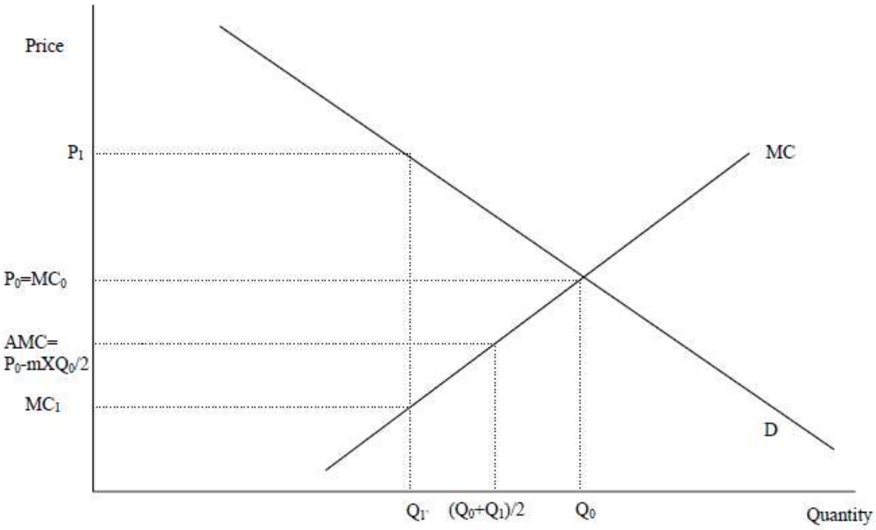
**Equation 76 - Critical loss - more generalised function**

$$x = \frac{y}{y + AMCM}$$

**Source:** Coate & Williams (2005, p. 10)

Consequently, the authors consider correct Harris and Simons' model as long as the "average marginal contribution" is used in place of the average marginal contribution margin in calculating x. To clarify the example, the authors referred to a homogeneous good with a linear marginal cost curve, as observed below:

**Figure 10 - Illustrating the average marginal cost concept**




**Source:** Coate & Williams (2005, p. 11)

Since the marginal cost is a linear curve, we have:

**Equation 77 - Average marginal cost (AMC) between and**

$$AMC = P_0 - \frac{mxQ_0}{2}$$

**Source:** Coate & Williams (2005, p. 11)

**Equation 78 - Average marginal contribution**

$$AMCM = \frac{P_0 - AMC}{P_0} = \frac{P_0 - \left[P_0 - \frac{mxQ_0}{2}\right]}{P_0} = \frac{mXQ_0}{2P_0} = \frac{x\varepsilon_{mc}^0}{2}$$

**Source:** Coate & Williams (2005, p. 11)

**Equation 79 - New critical loss**

$$\varepsilon_{mc}^0 = elasticity\ of\ the\ marginal\ cost\ curve\ in\ Q_0$$

$$x = \frac{\boldsymbol{y}}{\boldsymbol{y} + \frac{x\varepsilon_{mc}^0}{2}}$$

**Source:** Coate & Williams (2005, p. 12)

**Equation 80 - New critical loss expressed in a different manner**

$$\frac{x^2 \varepsilon_{mc}^0}{2} + xy - y = 0$$

**Source:** Coate & Williams (2005, p. 11)

**Equation 81 - The quadratic formula of the new critical loss**

$$x = \frac{\sqrt{y(y + 2\varepsilon_{mc}^0)} - y}{\varepsilon_{mc}^0}$$

**Source:** Coate & Williams (2005, p. 11)

**Table 13 - Generalised critical loss**

Critical Loss for Linear Marginal Cost Curve

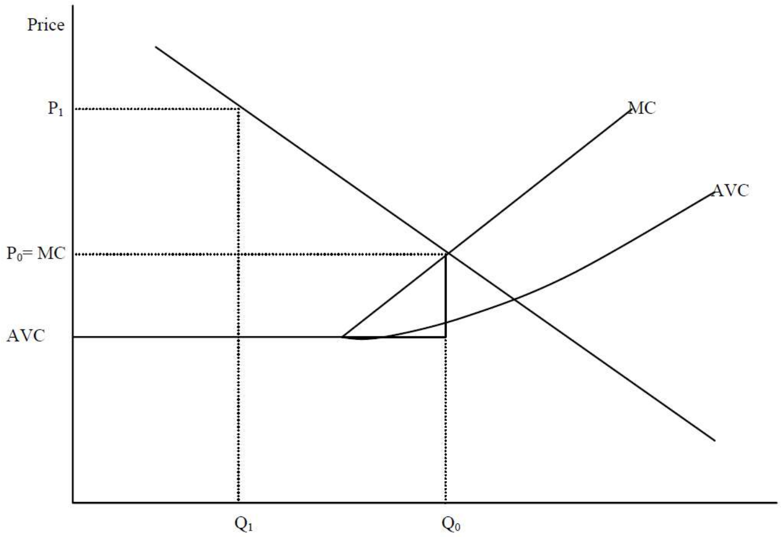
| Elasticity of marginal cost | Price Increase | | | | |
|---|---|---|---|---|---|
| | 5.0% | 10.0% | 15.0% | 20.0% | 25.0% |
| 0.01 | 91.6% | 95.4% | 96.9% | 97.6% | 98.1% |
| 0.05 | 73.2% | 82.8% | 87.3% | 89.9% | 91.6% |
| 0.1 | 61.8% | 73.2% | 79.1% | 82.8% | 85.4% |
| 0.2 | 50.0% | 61.8% | 68.6% | 73.2% | 76.6% |
| 0.25 | 46.3% | 58.0% | 64.9% | 69.7% | 73.2% |
| 0.5 | 35.8% | 46.3% | 53.1% | 58.0% | 61.8% |
| 1 | 27.0% | 35.8% | 41.8% | 46.3% | 50.0% |
| 2 | 20.0% | 27.0% | 31.9% | 35.8% | 39.0% |
| 4 | 14.6% | 20.0% | 23.9% | N/A | N/A |
| 5 | 13.2% | 18.1% | N/A | N/A | N/A |
| 10 | 9.5% | N/A | N/A | N/A | N/A |
| 20 | N/A | N/A | N/A | N/A | N/A |
| 100 | N/A | N/A | N/A | N/A | N/A |
| 1000 | N/A | N/A | N/A | N/A | N/A |

(COATE & WILLIAMS, Generalized Critical Loss for Market Definition, 2005, p. 13)

According to the authors:

> *"This analysis defines the general critical loss in a homogeneous market with linear marginal costs and the assumption that premerger prices equate to marginal cost at the competitive level of output. In some cases (marked N/A), the required critical loss would push the linear marginal cost function below zero and hence the hypothetical calculation is not valid. This negative marginal cost problem suggests that a more sophisticated cost model is required to create a generalized critical loss structure. Such a model would allow marginal cost to equal average variable cost for a range of output values but then increase linearly [after the said set] so the standard market equilibrium could be generated." (COATE & WILLIAMS, Generalized Critical Loss for Market Definition, 2005, p. 13)*

**Figure 11 - Proposed model**




Then, their suggestion is to calculate the critical loss as follows:

**Equation 82 - Generalised critical loss of break-even**

$$\boldsymbol{x} = \frac{2\varepsilon_{mc}^{0}y + AVCM^{2}}{2\varepsilon_{mc}^{0}(AVCM + y)}$$

**Table 14 - Generalised critical loss with a 5% price increase**

| Price Increase | 5.0% | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Variable Contribution Margin | | | | | | | | |
| Elasticity of marginal cost | 10.0% | 20.0% | 30.0% | 40.0% | 50.0% | 60.0% | 70.0% | 80.0% | 90.0% |
| 0.01 | 366.7% | 820.0% | 1300.0% | 1788.9% | 2281.8% | 2776.9% | 3273.3% | 3770.6% | 4268.4% |
| 0.05 | 100.0% | 180.0% | 271.4% | 366.7% | 463.6% | 561.5% | 660.0% | 758.8% | 857.9% |
| 0.1 | 66.7% | 100.0% | 142.9% | 188.9% | 236.4% | 284.6% | 333.3% | 382.4% | 431.6% |
| 0.2 | 50.0% | 60.0% | 78.6% | 100.0% | 122.7% | 146.2% | 170.0% | 194.1% | 218.4% |
| 0.25 | 46.7% | 52.0% | 65.7% | 82.2% | 100.0% | 118.5% | 137.3% | 156.5% | 175.8% |
| 0.5 | 40.0% | 36.0% | 40.0% | 46.7% | 54.5% | 63.1% | 72.0% | 81.2% | 90.5% |
| 1 | 36.7% | 28.0% | 27.1% | 28.9% | 31.8% | 35.4% | 39.3% | 43.5% | 47.9% |
| 2 | 35.0% | 24.0% | 20.7% | 20.0% | 20.5% | 21.5% | 23.0% | 24.7% | 26.6% |
| 4 | 34.2% | 22.0% | 17.5% | 15.6% | 14.8% | 14.6% | 14.8% | 15.3% | 15.9% |
| 5 | 34.0% | 21.6% | 16.9% | 14.7% | 13.6% | 13.2% | 13.2% | 13.4% | 13.8% |
| 10 | 33.7% | 20.8% | 15.6% | 12.9% | 11.4% | 10.5% | 9.9% | 9.6% | 9.5% |
| 20 | 33.5% | 20.4% | 14.9% | 12.0% | 10.2% | 9.1% | 8.3% | 7.8% | 7.4% |
| 100 | 33.4% | 20.1% | 14.4% | 11.3% | 9.3% | 8.0% | 7.0% | 6.3% | 5.7% |
| 1000 | 33.3% | 20.0% | 14.3% | 11.1% | 9.1% | 7.7% | 6.7% | 5.9% | 5.3% |

(COATE & WILLIAMS, Generalized Critical Loss for Market Definition, 2005, p. 16)

These calculations were elaborated for cases involving homogeneous goods. That is, this analysis has yet to be adapted to be useful for heterogeneous goods. Moreover, this model expects that costs will rise as the quantity increases. Such an assumption may not be realistic in some markets.

### ii. Generalised Critical Loss [profit maximisation - linear]

In a similar critique to Coate and Williams (as acknowledged by COATE & SIMONS, 2009) and aligned with the criticism of the AVC as a proxy, Baumann and Godek suggested using a new value of critical elasticity. However, the authors focused on linear profit maximisation instead of analysing the break-even from a perspective they believe to be more realistic (i.e. that marginal costs rise) in place of the former approach that considered marginal costs constant (and that used a proxy that did not match the reality):

**Equation 83 – Critical elasticity of profit maximisation in a linear demand**

$$\varepsilon_{critical} = \frac{-1 + \sqrt{1 + \frac{2m}{x}}}{2m}$$

**Source:** Baumann & Godek (2009)

**Table 15 - Critical elasticities according to Baumann and Godek**

| Critical Elasticities | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Assumption* | *Price Increase* | *Variable Cost Margin* | | | | | | | | |
| | | *10%* | *20%* | *30%* | *40%* | *50%* | *60%* | *70%* | *80%* | *90%* |
| *Constant Marginal Cost* | *5%* | 5.0 | 3.3 | 2.5 | 2.0 | 1.7 | 1.4 | 1.3 | 1.1 | 1.0 |
| | *10%* | 3.3 | 2.5 | 2.0 | 1.7 | 1.4 | 1.3 | 1.1 | 1.0 | 0.9 |
| | *15%* | 2.5 | 2.0 | 1.7 | 1.4 | 1.3 | 1.1 | 1.0 | 0.9 | 0.8 |
| *Increasing Marginal Cost* | *5%* | 6.2 | 5.0 | 4.3 | 3.9 | 3.6 | 3.3 | 3.1 | 3.0 | 2.8 |
| | *10%* | 3.7 | 3.1 | 2.7 | 2.5 | 2.3 | 2.2 | 2.1 | 2.0 | 1.9 |
| | *15%* | 2.6 | 2.3 | 2.1 | 1.9 | 1.8 | 1.7 | 1.6 | 1.5 | 1.4 |

**Source:** Baumann & Godek (2009, p. 7)